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(1) SECTION 3.4; Page 218; PROBLEMS: 3, 9, 13, 15
AVERAGE RATE OF CHANGE:
Be able to calculate average rate of change of a function over a given interval with the function given in table form, graph, or in function notation.
Remember that average rate of change of a function $f$ over an interval $[a, b]$ is found the same way that slope of a line is found: "The change in $y$" divided by "The change in $x$."
So average rate of change of a function $f$ over an interval $[a, b]$ is $\frac{f(b) - f(a)}{b - a}$

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(2) SECTION 3.5; Page 238; PROBLEMS: #15, 19, 23, 59 - 64
UNDERSTANDING AND ESTIMATING DERIVATIVES
Since the "Table" problems in this section are rather messy calculations, I will not ask those on the Final.
You should be able to do problems that involve looking at a graph and determining characteristics of the derivative (#15, 19, 23) or the correlation between the graph of the function and the graph of the derivative (#59 - 64)

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(3) SECTION 3.6; Page 253; PROBLEMS: #15, 17, 25, 39
THE DERIVATIVE
There will be one problem where you will be asked to find $f'(x)$ using the definition of the derivative. (What the book calls algebraically.) For this you must find and evaluate:

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Keep in mind we are taking secant lines between the points $(x, f(x))$ and $(x+h, f(x+h))$ and looking at their slopes. As $h$ "gets close to" 0, $x + h$ "gets close to" $x$ and the secant lines limit on the tangent line. Remember another interpretation of this process involves average rate of change and instantaneous rate of change.
You will be expected to be able to find the equation of a tangent line at a given $x$-value (#39); however, for that problem, you should find $f'(x)$ the "short way".

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(4) SECTION 3.7; PAGE 265; PROBLEMS 17, 27, 31, 35, 39, 53, 61, 65, 69, 93, 99
SIMPLE DERIVATIVES THE SHORT WAY:

BASIC POWER RULE: If $f(x) = x^n$, $f'(x) = nx^{n-1}$

Constant multiples and sums work in "the natural way" as you should be very familiar. Remember to be able to give your answer with NO NEGATIVE OR FRACTIONAL EXPONENTS. (It is not necessary to rationalize denominators in algebraic expressions.) Notice problems here that "look like quotients" (#27, 31, 39) but can be worked more easily by changing to negative exponents and using the simple power rule. Notice in #39 it is easier to rewrite by breaking the fraction up to the difference of two fractions with the denominator $x$, and simplifying. I will demonstrate the similar idea from #40:

$t(x) = \frac{2x + x^2}{x}$

$t(x) = \frac{2x}{x} + \frac{x^2}{x}$

$t(x) = 2 + x$ Now it's easy to take the derivative.

Also notice you will need to change radicals to fractional exponents (#35).
Be able to find the slope of $f$ at an $x$-value and the equation of the tangent line at that point. (#53, 61)
Also be able to find $x$-values where the function has horizontal tangents. (These are the same as the $x$-values for "stationary points" and are found by setting $f'(x) = 0$ and solving for $x$.) (#65, 69)

APPLICATIONS: Remember derivatives can be used to find any instantaneous rate of change. Here are the situations of rate of increase in the number of soccer teams (#93) and instantaneous velocity (rate of change of distance, $s$ (#99))

(5) SECTION 3.8; PAGE 276; PROBLEMS: #9, 11, 13, 15
APPLICATIONS (See NOTE at the end of the guide.)
(See Notes given in class on Marginal Analysis)
Remember Marginal Cost is found by taking the derivative of the total cost function $C(x)$. Marginal cost at $x = a$ is the approximate cost of producing unit $a + 1$. This can be compared to the actual cost of producing unit $a + 1$ by evaluating $C(a + 1) - C(a)$. Marginal Cost can also be thought of as how fast the cost is increasing (or decreasing.)

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(6) SECTION 4.1; PAGE 304; PROBLEMS: 23, 27, 37, 43, 63, 67
DERIVATIVES OF PRODUCTS AND QUOTIENTS

PRODUCT RULE $y = f(x)g(x)$ $y' = f'(x)g(x) + f(x)g'(x)$

QUOTIENT RULE $y = \dfrac{f(x)}{g(x)}$ $y' = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

CAUTION: Remember that unless the product involved is *only* a constant multiple (such as $5x^2$), you MUST use the product rule. (#23, 27) Be able to identify when this will be necessary in later sections, especially where an $e^x$ or ln function is involved and with implicit differentiation and partial differentiation.
Likewise for quotients, although sometimes the problem here is using it when the power chain rule would be easier. (For example, #19 in 4.2) Be able to simplify $f'(x)$ and find the equation of the tangent line at a given $x$-value (#63, 67)

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(7) SECTION 4.2; PAGE 316; PROBLEMS 13, 19, 25, 33, 39
CHAIN RULE: POWER FORM

$y = [f(x)]^n$ $y' = n[f(x)]^{n-1}f'(x)$

Constant multiples still "just follow through differentiation" Notice uses of the Chain Rule for radicals involving more than just $x$ (#25), and where you have a quotient, but the numerator is constant, so the quotient rule can be avoided (#19). Be able to simplify $f'(x)$.

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(8) SECTION 4.3; PAGE 327; PROBLEMS: 9, 17, 19, 23, 25, 27, 29, 43, 53, 55, 57, 65.
DERIVATIVES OF LOGARITHMIC AND EXPONENTIAL FUNCTIONS

If $y = e^x$, $y' = e^x$

If $y = \ln x$, $y' = \dfrac{1}{x}$

When taking derivatives involving $e^x$ and $\ln x$, our previous rules about constants, sums, products, quotients, and powers still apply. Be cautious mainly in recognizing products (#17, 55), quotients (#65), powers of functions (#57), and correct application of chain rule. (#25, 29, 55)

CHAIN RULE: GENERAL FORM
Though we saw the truly general form of the chain rule here, we can restrict ourselves to the cases for $[f(x)]^n$, $e^{f(x)}$ and $\ln f(x)$:

1. If $y = [f(x)]^n$, then $y' = n[f(x)]^{n-1}f'(x)$ (This was covered in Section 4.2)

2. If $y = e^{f(x)}$, then $y' = e^{f(x)}f'(x)$

3. If $y = \ln f(x)$, then $y' = \dfrac{1}{f(x)}f'(x)$ or $y' = \dfrac{f'(x)}{f(x)}$

Here again be cautious to recognize products, quotients, chain rules and combinations of these.

(9) SECTION 4.4; PAGE 337; PROBLEMS: 3, 5, 7, 13, 15, 21
IMPLICIT DIFFERENTIATION
Remember here we are working with equations that have 2 or more variables. We assume that one of the variables is a function of the other, (For example, $y$ is treated as a function of $x$ in #1-10); or that they are both functions of a third variable (See 5.4 on Related Rates.) With this assumption, finding $y'$ or $\frac{dy}{dx}$ involves differentiating each term of the equation, being careful to recognize products and chain rules. Once the first step has been completed, all terms that contain $\frac{dy}{dx}$ can be shifted to one side of the equation and all terms that do not contain $\frac{dy}{dx}$ can be shifted to the other side. Since all terms on one side contain $\frac{dy}{dx}$, then $\frac{dy}{dx}$ can be factored out as a common factor. This leads to a solution for $\frac{dy}{dx}$ by dividing both sides of the equation by the factor in ( ). If asked to evalute $\frac{dy}{dx}$ at a given point, then you just substitute the appropriate $x$ and $y$ values into the expression you have found for $\frac{dy}{dx}$.

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(10) SECTION 5.1; PAGE 361; PROBLEMS 1, 3, 7, 9, 11, 15, 17, 19, 23
OPTIMIZATION: ABSOLUTE MAXIMA AND MINIMA
Just as the term implies, an absolute max. is a value of the function that is greater than every other value of the function. (Correspondingly for an absolute min.) There are some functions such as parabolas that may have one absolute extreme without restricting the values of $x$; however, we often only need to look at the function restricted to either an open or closed interval of values for $x$.

In the case of restricting the values of $x$ to a closed interval $[a, b]$ (that is $a \leq x \leq b$), in order to find the absolute extrema, we need to check the values of the function at the endpoints (that is $f(a)$ and $f(b)$) and for any critical values $c$ in the interval $[a, b]$ (where $f'(c) = 0$) you must also find $f(c)$. Now compare all of these "$y$"-values. The largest one is the absolute maximium and the smallest is the absolute minimum.

USES OF THE FIRST DERIVATIVE IN GRAPHS

FROM THE GRAPH: See Figures 1 - 3 on page 350. Be able to:
Determine the intervals where $f$ is increasing (same as where $f'(x) > 0$)
Determine the intervals where $f$ is decreasing (same as where $f'(x) < 0$)
Determine where there are horizontal tangents ($f'(x) = 0$)
Determine where $f'(x)$ does not exist either because there is a vertical tangent (at $x = k$ in the figure) or by a sudden change in tangent lines (at the point $(2, -2)$ in problem #10 on page 361)
Also, find the points for relative and absolute maxima and minima. (Remember that local extrema can occur where $f'(x) = 0$ or where $f'(x)$ does not exist, assuming $f(x)$ exists at that point.)

FROM THE FUNCTION: Be able to find all the things mentioned above, working from the function, rather than a graph. Remember you will be using techniques for solving non-linear inequalities since you will be wanting to solve $f'(x) > 0$ and $f'(x) < 0$.
It is good here to create a "Number Line Chart" for each of $f$ and $f'$, indicating where they are positive and negative. Where $f$ is positive, the graph is above the $x$-axis, where $f$ is negative, the graph is below the $x$-axis.
Where $f'$ is positive, the graph is increasing, which can be indicated on the chart by a line sloping up (positive slope) left to right. Where $f'$ is negative, the graph is decreasing, which can be indicated in a similar way by a negative sloping line. Practice graphing, using the information about increasing, decreasing and local extrema.

(11) SECTION 5.2; PAGE 370; PROBLEMS 1, 3, 5
OPTIMIZING FUNCTIONS WITH CONSTRAINTS:

Here you are asked to Minimize or Maximize an object function that may be expressed in terms of more than one variable. There are "constraint" equation(s) and/or bounds stated for the variables involved. Generally, you can use the constraint equation to solve for one of the variables in terms of the other and substitute this expression into the object function in order to be able to express it in terms of one variable only. Once this is done, we can use our previous understanding of optimizing functions to take the derivative, etc. Notice any bounds that are expressed for the variables such as $x \geq 0$ that will affect the interval over which you are optimizing the function.
REMEMBER for all problems in finding abs. max or min. to show by a Number line for $f'$ that the value does yield the desired max or min.

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(12) SECTION 5.3; PAGE 385; PROBLEMS 1, 7, 9, 17-31, 33, 35, 39
USES OF THE SECOND DERIVATIVE IN GRAPHS

FROM A GRAPH of $f$: See problems #17-31.Be able to:
Identify intervals where $f$ is concave up (same as $f''(x) > 0$)
Identify intervals where $f$ is concave down (same as$f''(x) < 0$)
Identify inflection points. (you will see the change from concave up to down or vice versa.) Problems #17-23)
ALSO be able to identify inflection points of $f$ if you are given the graph of $f$ (Problems 17-24), $f'$ (Problems 25-28) or $f''$ (Problems 29-32.)
From GRAPH of $f'$:Inflection points for $f$ will come where $f''$ changes from + to - (or vice versa). Since we are looking at $f'$, we want the derivative of $f'$ (which is $f''$) to change sign. The derivative of any function is positive when the function is increasing, so look for sections where $f'$ is INCREASING, and those will be concave UP for $f$. Similarly, look at sections where $f'$ is DECREASING and those will be concave DOWN for $f$. If a function changes from increasing to decreasing, there is a relative max, and if the function changes from decreasing to increasing then there is a relative min. These changes in $f'$ thus yield an INFLECTION POINT in $f$ where there is a REL. MAX or MIN in $f'$. We can only determine the $x$-value of the point in $f$ since we are looking only at the graph of $f'$.
From GRAPH of $f''$: Since we need the sign of $f''$ to change to create an inflection point, we need to look at where $f''$ is positive and where $f''$ is negative. A function is positive in the sections where it is ABOVE the $x$-axis, and negative where it is BELOW the $x$-axis. The $x$-value where it changes from above to below, or vice versa must either be an $x$-intercept or a vertical asymptote. If you were told that the original function $f$ was defined for all real numbers and continuous, then any asymptote seen in $f'$ or $f''$ is indicating that the derivative(s) are undefined, NOT that the function is undefined. Therefore, there is a point with that $x$-value. The $x$-values for $x$-intercepts and for vertical asymptotes will thus be the $x$-values for inflection points in $f$.
You may also refer to Test #2, problems 12 - 14.

FROM A FUNCTION $f$: Be able to find $f''(x)$ (#1, 7, 9)

Find intervals where the graph is concave up, where it is concave down, and inflection points (#33, 35, 39). Again you will possibly be using techniques of solving non-linear inequalities, since you will be solving $f''(x) > 0$ and $f''(x) < 0$. You will also want to make a Number Line Chart for this, labeling it as $f''$. In #33 $f''$ is linear, but the situation in #35 is more typical. It is helpful to indicate the concave up and down shapes over the correct areas of the number line chart.
Remember $c$ is an $x$-value of an Inflection Point $f''(c) = 0$ or $f''(c)$ is undefined AND is found where $f''(x)$ CHANGES from $+$ to $-$ or vice versa on either side of $c$. Obviously $f(c)$ must exist for this to be an inflection POINT. Answer should thus be in the form $(c, f(c))$, a *point* of the function.

GRAPHING FROM THE INFORMATION: I will give you the Number Line Charts indicating where each of $f$, $f'$, and $f''$ are 0, undefined, positive, and negative. There will also be an $x$, $f(x)$ chart to show the $y$-values for all important points ($y$-intercept, local extrema and inflection points). Any Verical or Horizontal Asymptotes will also be indicated. You should be able to accurately graph with correct shape for concavity, horizontal, vertical tangents, etc. (See #16 on MT#2 and #7 on MT#2, Take-Home Part. Also see WS#2).

GRAPHING FROM THE FUNCTION: (#37, 39, #6 on MT#2 Take-home part) If I ask you to graph the function "from scratch", it will be a "basic polynomial" with instructions similar to the cited problem from Test #2, Take-home. Alternately, I might ask you NOT to graph BUT to set up ALL the Number Lines with the signs interpreted correctly for above & below; increasing, decreasing; concave up & down. From either set up, be able to identify $x$-intercepts, rel max & rel min, and points of inflection.

QUESTIONS RELATED TO GRAPHING PROBLEMS, BUT NOT NECESSARILY ASKED TO GRAPH: You should be able to set up any number lines for $f$, $f'$, or $f''$ with the appropriate results as we have shown in class to indicate above and below the $x$-axis, increasing and decreasing intervals, and intervals where it is concave up and concave down. Also be able to draw conclusions about local and absolute extremes, and inflection points that are indicated from $f'$ and $f''$ number lines.

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(13) SECTION 5.4; PAGE 396; PROBLEMS: 9, 11, 13; from Test #2: #6 and 7
RELATED RATES
For these problems, we are dealing with equations of 2 or more variables, where each variable is assumed to be a function of time, $t$. There are many applications of this idea, but there will either be an equation given that applies to the problem or it will be a well-known geometric formula such as the pythagorean theorem that relates the parts being discussed. In setting up these problems, DRAW a PICTURE and label. For example, in the case of a ladder leaning against a wall, you should see the idea of a right triangle being formed by the wall, the ground and the ladder. Label any side that is NOT fixed as some variable, such as $x$ or $y$. If the side is constant for the entire problem, such as the length of the ladder, that constant can be used even at the beginning.
CAUTION: Do NOT use any length, etc. that can change during the problem, until AFTER YOU HAVE TAKEN THE DERIVATIVE with respect to $t$. Determine from the figure what rates you are given (such as $\frac{dx}{dt}$) and which rates you are trying to find (such as $\frac{dy}{dt}$). After using implict differentiation with respect to $t$, NOW SUBSTITUTE the known values of rates and the lengths that were variable during the problem and solve for the needed rate.

(14) SECTION 6.1; PAGE 428; PROBLEMS: 3, 5, 7, 11, 17, 31, 33, 39, 45, 47, 51, 53
ANTIDERIVATIVES AND INDEFINITE INTEGRALS

Now we are reversing the process.... We have to work backwards from what is assumed to be the derivative to the function that it "came from". Since constant multiples "go through" the differentiation process unchanged, we would expect the same when the process is reversed. Likewise for sums. Here are reminders for other basic indefinite integrals:

In the following $k$ and $C$ are constants:

1. $\displaystyle\int k\,dx = kx + C$

2. $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ if $n$ is NOT $-1$ (see 4. below) or $\displaystyle\int x^n\,dx = \frac{1}{n+1}x^{n+1} + C$

This can be said as increase the exponent by 1 and multiply by the *inverse* of the result. The idea of inverse is particularly useful when dealing with fractional exponents. Note that if $n = -1$ and we try to use this rule, we run into an undefined expression since $n + 1 = 0$ is in the denominator.

3. $\displaystyle\int e^x\,dx = e^x + C$

4. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C$ This takes care of our situation in 2. of $n = -1$, since another way of stating 4. is:

4. $\displaystyle\int x^{-1}\,dx = \ln|x| + C$

COMMENTS: Remember the $dx$ in the integral expression is indicating to you to take the *integral* "with respect to" $x$, just as $\frac{dy}{dx}$ indicated to take the *derivative* with respect to $x$. In these exercises, it seems the natural thing to do, and we can tend to ignore $dx$, but remember from our later work with partial derivatives and their corresponding integrals that it is necessary to indicate what variable we are considering.

CAUTION: The $dx$ and $\int$ sign should be gone as soon as you have integrated. (Leaving one or both of these in the integral expression would be like saying $\sqrt{25} = \sqrt{5}$)

REMEMBER the "trick" from 3.7 about rewriting radicals and constants over variables to powers as constants times variables to negative exponents. Also remember that fractions over a monomial denominator can be "broken apart" using the idea that $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ (#39).

CAUTION: The integral of a product is NOT the product of the integrals. This should make sense because we know that the derivative of a product is NOT the product of the derivatives. Hence, if possible when a product is involved in an integral, MULTIPLY OUT FIRST. In some situations this is impractical or impossible, so in 6.2 we will see some products that we can integrate and why.

(15) SECTION 6.2; PAGE 437; PROBLEMS: 11, 17, 21, 27, 32, 33
INTEGRATION BY SUBSTITUTION (OR REVERSE THE CHAIN RULE)
These problems can either be done by a substitution, letting $u =$ and finding $du =$ $dx$ so that substitutions can be made into the original integral so that in the variable $u$, the integral now looks like 1., 2., 3., or 4. from 6.1 above.
Since $x$ is a "dummy variable," our list of antiderivative forms in (14) can be listed using $u$.

1. $\int k\ du = ku + C$
2. $\int u^n\ du = \frac{u^{n+1}}{n+1} + C$ if $n$ is NOT $-1$ (see 4. below) or $\int u^n\ du = \frac{1}{n+1}u^{n+1} + C$
3. $\int e^u\ du = e^u + C$
4. $\int \frac{1}{u}\ du = \ln|u| + C$ This takes care of our situation in 2. of $n = -1$, since another way of stating 4. is:
4. $\int u^{-1}\ du = \ln|u| + C$

When using substitution, the integral should "look like" one of these forms after substitution. There can be NO other $u$'s or $x$'s involved. There can, of course, be constant multiples, which we know just "follow through" the integral process. Be sure to keep up with these constants.

For example: $\int 5x(x^2+1)^4\ dx$ . Let $u = x^2 + 1$, then $du = 2x\ dx$, so $dx = \frac{1}{2x}du$

Substituting, we get: $\int 5xu^4 \left(\frac{1}{2x}\right)\ du$.
Notice that cancelling the $x$'s and multiplying the constants together leaves us with

$\int \frac{5}{2}u^4\ du$ This is in the form of 2. above with a constant multiple, so we get:

$$\left(\frac{5}{2}\right)\frac{u^{4+1}}{4+1} + C = \frac{5u^5}{10} + C = \frac{(x^2+1)^5}{2} + C$$

Alternately, with the same substitution, you may solve for $xdx = \frac{1}{2}du$. Substituting $\frac{1}{2}du$ for $xdx$ also yields the following:
$\int \frac{5}{2}u^4\ du$ and proceed as above.

ALTERNATELY: You can visualize that your expression is in the *form* $\int [f(x)]^n f'(x)$ and mentally realize that going backward through the chain rule that $f'(x)$ will "drop out" and $[f(x)]^n$ will obey the basic power rule in integrating of "add one to the exponent and divide by the resulting new exponent."

SIMILARLY: Recognize that in $\int e^{f(x)} f'(x)\ dx$, $e^{f(x)} f'(x)$ is the result of the chain rule in 4.3, 2.

ALSO: In $\int \frac{f'(x)}{f(x)}\ dx$, $\frac{f'(x)}{f(x)}$ is the result of the chain rule in 4.3, 3.

NOTE: this could also look like: $\int \frac{1}{f(x)} f'(x)\ dx$

FIXING IT UP: Remember with this latter "visualizing method" that it will often be necessary to multiply by a *constant* to get the $f'(x)$ part to be correct. This can be legally done by multiplying by the necessary *constant* on the inside of the integral to get the correct form and then on the outside to compensate by multiplying by the inverse of this *constant*. (Thus you have multiplied by ONE and not changed the problem's value.) CAUTION: This can only be done with CONSTANTS.

(16) SECTION 6.3; PAGE 452; PROBLEMS: 19, 25, 29, 31

UNDERSTANDING DEFINITE INTEGRALS AS AREAS USING GRAPHS AND GEOMETRY.

Here we are interpreting the idea of the definite integral by looking at examples that we can interpret geometrically. Remember that the idea is that the definite integral of a function over an interval where the function lies entirely above the $x$-axis is equal to the area under the curve (the area created by the function and the $x$-axis.) Also, the definite integral of a function over an interval where the function lies entirely *below* the $x$-axis is equal to the *negative* of the *area* created by the function, and the $x$-axis. From these two facts we can see that if we evaluate a definite integral of a function over an interval where part of the function is above the $x$-axis and part of it is below the $x$-axis, then there will essentially be positive and negatives adding together. Keep this in mind when working these geometric examples.

#19-25 require graphing the desired function first, which is linear in each of these examples.

In #29-33, the graph is already in place. In both types, you interpret what would happen to the definite integral considering "positive" and "negative" areas being combined. If asked to do so using Geometry, remember to show work that indicates what geometric formula was used, considering the geometric shape that the graph indicates.

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(17) SECTION 6.4; PAGE 464; PROBLEMS: 1, 3, 8, 11, 17, 25, 51, 53, 57

DEFINITE INTEGRALS USING ANTIDERIVATIVES

This section ties in the antiderivative to the idea of the area under a curve. If you are ever asked about the

FUNDAMENTAL THEOREM OF CALCULUS, this is it:

If $f$ is a continuous function on the closed interval $[a, b]$ and $F$ is any antiderivative of f, $(F'(x) = f(x))$ then

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b = F(b) - F(a)$$

In the case that $f(x) \geq 0$ over the interval $[a, b]$ (i.e., $f(x)$ lies *above* the $x$-axis over this interval, the value found corresponds to the AREA UNDER THE CURVE FROM $a$ to $b$. (If the function lies totally *below* the $x$-axis over this interval, the value found will be the $NEGATIVE$ of the AREA.

Notice if the areas lie below *and* above the $x$-axis over the indicated interval, that the area below the $x$-axis gets subtracted from the area above the $x$-axis.

NOTE: Since you will not be using calculators, if the calculation of a definite integral is "messy", I will just have you set up the antiderivative to be evaluated. In that case I mean for you to leave the answer at the $F(x)\Big|_a^b$ step

(18) SECTION 7.2; PAGE 491; PROBLEMS: 3, 5, 11,
AREA BETWEEN CURVES
We set up the basis for this work in 6.4. There we saw how to find areas over intervals that were either totally above or totally below the $x$-axis. If you want to find areas that lie above and below the $x$-axis, it is necessary to "break the areas" into parts that lie totally above *plus* the negative of (or absolute value of) those that lie totally below the $x$-axis. It will usually be necessary to sketch a graph, finding the $x$-intercepts to determine where the desired area is and how to break it up properly.

FOR EXAMPLE: Find the area enclosed by $y = -2x - 1$ and $y = 0$ over the interval [0,4]. $y = -2x - 1$ is a line with $y$-intercept $-1$ and slope $-2$. It has $x$-intercept $-\frac{1}{2}$ or $-0.5$, so over the interval $0 \leq x \leq 4$, the area bounded by this line and $y = 0$ (the $x$-axis) lies totally below the $x$-axis, thus the desired area is found by evaluating the following definite integral:

$$-\int_0^4 (-2x - 1)\, dx$$

However, notice if the interval had been changed to $-3 \leq x \leq 4$, we would have to do the following to find the desired area:

$$\int_{-3}^{-0.5} (-2x - 1)\, dx - \int_{-0.5}^{4} (-2x - 1)\, dx$$

This is necessary because over the interval $[-3, -0.5]$, the line is above the $x$-axis and over the interval $[-0.5, 4]$, the line is below the $x$-axis.

IN GENERAL: If you are trying to find the area between 2 curves $f(x)$ and $g(x)$, over an interval $[a, b]$, and $f(x) \geq g(x)$ for every $x$ in this interval (That is over the desired interval, $f(x)$ lies *above* $g(x)$, then the:
AREA BETWEEN THE CURVES $f(x)$ and $g(x)$ over the interval $[a, b]$ is

$$\int_a^b [f(x) - g(x)]\, dx$$ (#11) See Also: MT#3, problem 17 and Take Home Part, problem 7.

(19) SECTION 8.1
FUNCTIONS OF SEVERAL VARIABLES
No separate problems directly from this section. Just understand how to evaluate multi-variable functions as needed later.

(20) SECTION 8.3; PAGE 566; PROBLEMS: 3, 7, 9, 11, 13
PARTIAL DERIVATIVES
This may look like the situation for implicit differentiation at first, so carefully read directions and keep track of notation. We are working with equations that have more than one variable, but in this case, we are treating *all but one* of the variables as a CONSTANT. We do follow a pattern similar to implicit differentiation in looking at the equation term by term, just remember that ALL BUT THE INDICATED VARIABLE IS CONSTANT!

EXAMPLE: $C(x, y) = 5x^2 - 6xy + 3y^2 - 4x + 10y - 15$ We want to find $\frac{\partial C}{\partial y}$. (the Partial derivative of $C$ with respect to $y$.)
We are taking the partial "with respect to $y$" so that tells us to treat $x$ as a constant. Therefore:

$$\frac{\partial C}{\partial y} = 0 - 6x + 6y - 0 + 10 - 0$$
$$\frac{\partial C}{\partial y} = -6x + 6y + 10$$

CAUTION: Watch for products, quotients and chain rule.

(21) SECTION 8.5
MAX AND MIN USING LAGRANGE MULTIPLIERS
Due to limited time, I will not give a problem from this section.

(22) SECTION 8.6; PAGE 593; PROBLEMS 1, 3, 5, 37
USING INTEGRALS TO FIND VOLUMES UNDER SURFACES: DOUBLE INTEGRALS
This section ties together the reason for partial derivatives, partial antiderivatives, and extends the idea of the integral to find area to that of finding volume. There are many types of volumes that can be found with integrals, but this is perhaps the easiest. The idea is that instead of a curve $f(x)$ lying above the $x$-axis, we have a surface $f(x,y)$ lying above the $x,y$-plane.

VOLUME, V of the solid that is created "under" this surface over a rectangular box bordered by $a \le x \le b$ and $c \le y \le d$ is found by:

$$V = \int_c^d \left[ \int_a^b f(x,y)\,dx \right]\,dy$$

To evaluate this double integral, we follow the natural procedure of working "from the inside" with nested parentheses.

$\int_a^b f(x,y)\,dx$ means to find the partial antiderivative of $f(x,y)$ *with respect* to $x$, that is treat $y$ as a constant and integrate as usual.

Then for the limits of integration, we evaluate this partial antiderivative at $x = b$ MINUS the partial antiderivative evaluated at $x = a$.
Notice that you now have a function in the single variable $y$ to finish out the next integral.

(23) NOTE ON APPLICATION PROBLEMS: Under some of the application parts of sections, I have noted some types of problems that would be good to review. For some other applications, I note some problems in the notes below. Any problems given on the Final Exam will NOT require a calculator. Either I will make the arithmetic easy enough to avoid that, or I will allow you to leave your answer in an unsimplified form.
Basically, you need to be familiar with the following application concepts:
(1) "Marginal" cost, revenue, or profit function. (See Applications note to section 3.8)
(2) How to maximize or minimize any of these functions. (Section 5.1 and 5.2) Remember to indicate why your critical number yields the desired result (max or min).
(3) You should have worked with price, demand, cost , revenue, and profit enough by now to know the relationships between them. On the final, if you forget one of these, you may come and ask me... I won't count off much for the "hint"
(4) Remember if you are given a function representing Marginal Cost ($C'(x)$), Marginal Revenue ($R'(x)$), or Marginal Profit ($P'(x)$), the corresponding Cost, Revenue or Profit function will be the antiderivative (integral) of the marginal function. In order to find the *correct* antiderivative, you can be given a boundary condition such as fixed costs so that you can determine what "$+K$" is in the indefinite integral.
(5)To calculate total change in cost from a production level of $x = a$ units to $x = b$ units, you will need to evaluate the following definite integral: $\int_a^b C'(x)\,dx$ (Basically, the integral of marginal cost is total cost, so you are calculating the difference between total cost for $b$ units and total cost for $a$ units which is the desired total change in cost between these levels.)
Similarly, we can find total profit over a certain production level or over a certain time period.
(6)Also remember the relationships between position, velocity and acceleration.