COMPLEX NUMBERS ~~7.1~~

WHAT IS REAL?: Real numbers are the numbers that you have grown up with. You started with *counting* or *whole* numbers: 1, 2, 3, 4, ... Then you probably learned about fractions $\frac{1}{2}$, $\frac{3}{7}$, etc. These are called *rational* numbers which comes from the word *ratio* because they can be constructed from the *ratio* of two integers. Next, you probably learned about negative numbers (*Rational* numbers and *Integers* can be positive or negative.) Finally, you found out about some *irrational* numbers. These numbers, like $\sqrt{2}$, and π are *not* rational ... they can NOT be written as the ratio of 2 integers. They can be distinguished by the fact that their decimal expansions never repeat a pattern whereas a fraction like $\frac{1}{3} = .333333333333333333333333333333333333333333333333333333333$ (forever) is rational.
NUMBER LINE LINK: One way of thinking of Real Numbers is that they are all the possible, *tiny*, *infinitesimal* points on a number line. (No matter how close together you pick two points on the number line, there is always at least one more in between them.)

WHAT ISN'T REAL: The thing that you were probably told was impossible (and rightly so in the Real Numbers!) was to take the square root of a negative number. However, there is another whole system of numbers, that *contains* the Real Numbers. This system is the COMPLEX NUMBERS, and in the system of complex numbers *square roots* of *negative* numbers DO EXIST! But since they are NOT REAL NUMBERS, the basic one $\sqrt{-1}$ is defined as i for *imaginary*. (Yes, you can *claim* that mathematicians are now "making up" things to make trouble for you. However, there are actually many applications of Complex Numbers.)

COMPLEX NUMBER BASICS: The basic imaginary unit is i:

$\sqrt{-1} = i$ and $i^2 = -1$

$\frac{i}{i} = 1$ This is natural and useful in Simplification and in Rationalization.

WARNING: i does NOT equal -1

A complex number is any number that can be expressed in the form:
$a + bi$ where a and b are Real Numbers

Examples: $2 - 3i$, $\frac{3}{4} + 2i$, $i\sqrt{2}$ (Here, $a = 0$, and we write the i in front of the radical so that i does *not* appear to be *under* the radical, which would be incorrect.)

ADDING & SUBTRACTING COMPLEX NUMBERS is a very natural thing: You just combine *like* terms.
Example: $(2 + 4i) - (5 - 7i) = 2 + 4i - 5 + 7i = -3 + 11i$

WHAT WE WILL USE LATER: In solving certain equations later in the course, we will get answers that have square roots of negative numbers. We have to be able to correctly interpret these as Complex Numbers.
Example: $\sqrt{-18} = 3i\sqrt{2}$
Now keep in mind, in Notes #11 on Radicals, I mentioned to remember the restriction put on multiplying radicals together when we got to Complex Numbers.... Now is the time:

REMEMBER RESTRICTIONS: $\sqrt{A}\sqrt{B} = \sqrt{AB}$ ONLY if $A \geq 0$ and $B \geq 0$. So what do we do when A and/or B is negative? You use Order of Operations, which guides you to do what you can inside *parentheses* first. Oh, you don't see the parentheses? These are some of those "understood parentheses" .... Everything underneath a radical is "held together" by that radical just like parentheses do... So we would do what is under the radical first. Follow the Examples:
$\sqrt{-6}\sqrt{-8} = (i\sqrt{6})(i\sqrt{8}) = i^2\sqrt{48} = (-1)(4)\sqrt{3} = -4\sqrt{3}$ CORRECT!
BUT if we forgot the RESTRICTIONS above:
$\sqrt{-6}\sqrt{-8} = \sqrt{(-6)(-8)} = \sqrt{48} = 4\sqrt{3}$ WRONG!!!!!!!!!!.

WAY TO REMEMBER: Get the $i's$ out $first$, then you can deal with the other part of the radical problem in a normal manner... Just remember to simplify $i^2 = -1$. Also, I don't mind if you "skip steps", as long as your work is correct and logical, so that I can follow it. However, do not do everything on scratch paper and just write down an answer. If you skip a step, that is for something you did "in your head"; so I should be able to do the same.

MULTIPLYING WITH i: These problems are really done in a very natural manner. You just Distribute and FOIL as needed, and combine like terms. Just remember that $(i)(i) = i^2 = -1$, so that every term that is multiplied by i^2 has to be rewritten with the proper new sign and combined with other terms accordingly.
Example: $(3-2i)(4+5i) = 12 + 15i - 8i - 10i^2 = 12 + 7i - 10(-1) = 22 + 7i$

"RATIONALIZING DENOMINATORS": The book does not actually use the term "rationalize" here, but the idea is certainly fitting. The i represents the *square root* of -1 and since we are used to thinking of "getting rid of" radicals from the denominator, it is the same idea. Technically, we will be putting the number into that standard form $(a + bi)$. There are two types, just like there are two types of square roots that we have had to rationalize, and the methods are identical to the corresponding process in square roots. I will show an Example of each:

$$\frac{4}{5i} = \left(\frac{4}{5i}\right)\frac{i}{i} = \frac{4i}{5i^2} = \frac{4i}{-5} = -\frac{4i}{5}$$

$$\frac{2i}{5-3i} = \left(\frac{2i}{5-3i}\right)\left(\frac{5+3i}{5+3i}\right) = \frac{(2i)(5+3i)}{5^2-(3i)^2} = \frac{10i+6i^2}{25-9i^2} = \frac{10i-6}{25+9} = \frac{-6+10i}{34} = -\frac{6}{34} + \frac{10i}{34} = -\frac{3}{17} + \frac{5}{17}i$$

Notice that I made use of the CONJUGATE as I did in rationalizing binomial radicals. I made the final steps here to get the number simplified and into the $proper\ form$ of $a + bi$.