SOLVING EQUATIONS BY EXTRACTING ROOTS (7.2C)

NO THIS IS NOT A TRIP TO THE DENTIST!: Remember that *root* is another term for a solution to an equation. So we are going to be SOLVING certain kinds of equations. This book does not use the term *extract roots*, but it is used in other texts, and it is a very descriptive term as you will see.

BEGIN WITH A FAMILIAR EQUATION: If you were asked to SOLVE:
$x^2 - 9 = 0$ according to what we have discussed so far, you would naturally FACTOR:
$(x-3)(x+3) = 0$ and conclude by the Multiplicative Property of Zero that:
$x = 3 \quad x = -3$ are the two solutions or *roots*.

MAKE IT HARDER: Now change the equation *just slightly* to the following:
$x^2 - 8 = 0$ Although the number only changed from 9 to 8, the obvious problem is that we can not employ the factorization technique of the Difference of Squares since 8 is not a perfect square. Now let's go back to the original equation because we *know* what the roots are, and see if there is another technique we can employ that would bypass having to factor in this case. (No, it doesn't have to be as complicated as the *quadratic formula* here.) We have just solved equations where we *squared* both sides of the equation, so you may be anticipating the following:

$x^2 = 9$ First ISOLATE what I need to take the square root of.
WRONG STEP: $\sqrt{x^2} = \sqrt{9}$ Now what is the problem in this step? The RESULT of this is:
$x = 3$ And we have LOST a solution, namely $x = -3$.
WARNING: $\sqrt{9}$ does NOT equal ± 3 Just like any other "number," $\sqrt{9}$, is assumed to be positive unless a negative sign is put in front of it, so $\sqrt{9} = 3$ and $-\sqrt{9} = -3$.

SO....TO KEEP OUR EQUATIONS EQUIVALENT, WE MUST PUT IN THE $\pm$:
$\sqrt{x^2} = \pm\sqrt{9}$ CORRECT! Because now the result is:
$x = \pm 3$ This agrees with the solutions or *roots* that we obtained the first time by factoring.

So it becomes obvious to proceed with the *harder* problem, we would follow the same procedure:
$x^2 = 8$ First ISOLATE what I need to take the square root of.
$\sqrt{x^2} = \pm\sqrt{8}$ "SQUARE ROOT" BOTH SIDES & DON'T FORGET $\pm$ so equation is *equivalent*!
$x = \pm 2\sqrt{2}$ SIMPLIFY the RADICAL if possible.

REMEMBER: The symbol $\pm$ indicates TWO numbers: $\sqrt{8}$ and $-\sqrt{8}$ which simplify to $2\sqrt{2}$ and $-2\sqrt{2}$, respectively.
HOWEVER: It is NOT necessary to write them separately. ALL mathematicians will write this as $\pm 2\sqrt{2}$.
WARNING: In simplifying $\sqrt{8}$, remember that 2 is *multiplied* by $\sqrt{2}$, and that the $\pm$ just represents the two separate numbers as mentioned above, so do NOT simplify this as:
$2 \pm \sqrt{2}$ WRONG!!!!!!!!!!!!!!!!

GENERAL PATTERN TO EXTRACT ROOTS: In Algebra, we are constantly looking for patterns. Seeing the pattern is what can make the difference between an easy problem and an impossible one. In the following *pattern*, what you will have to learn is that A can be replaced by as large an expression as is needed, and that c can be any *constant*, that is, it is a number with NO variables in it. (The restriction on c is for our purposes this semester... NOT an *absolute rule*)
IF $A^2 = c$ then:
$\sqrt{A^2} = \pm\sqrt{c}$ so, finally:
$A = \pm\sqrt{c}$

EXPANDING OUR POSSIBLITIES: Now we are ready to try the following example...SOLVE:
$(x+5)^2 = 7$ Before our above discussion, your natural inclination with this problem likely would have been to FOIL, set it it equal to *zero* and try to factor.
BUT, you would have been doomed to failure, because the quadratic you would have obtained would NOT have factored. This is where you need to LOOK FOR THE PATTERN:
You have to picture $(x+5)$ as being the A in our General Pattern above. This allows us to:

$\sqrt{(x+5)^2} = \pm\sqrt{7}$ TAKE the SQUARE ROOT of BOTH SIDES; REMEMBERING $\pm$
$x+5 = \pm\sqrt{7}$ There is NO need to FOIL, the square root and the square essentially "cancel out." (This really makes sense now that we have the link of the radicals to the fractional exponents.)
$x = -5 \pm \sqrt{7}$ There are TWO numbers on the RIGHT, but we can work with them "together:" And we have our *roots* for the equation. Now you can see why you would not have been able to factor the corresponding quadratic, because we got irrational solutions.

NOTE: This solving terminology is doubly nice: It is a *root*, meaning solution, and we obtained it by taking or *extracting* a square *root*.

IMPORTANT STEPS TO REMEMBER: Not all of the problems will be "set up - ready to solve." So how do you know when to use this solving pattern rather than set equal to *zero*, factor and solve? Look for the following:

1. ONLY x is *squared*, there are no x-terms (of first degree), and there is a constant c in the equation. This is like our first two examples, so that we will be able to rearrange to get the form:
$x^2 = c$ And we can proceed as before.

2. A binomial *squared*, such as $(ax+b)^2$ appears in the equation with a and b being numbers, and there are "no other x's" in the equation. Then, we can again *make* this equation fit our pattern by ISOLATING $(ax+b)^2$ on one side of the equation and gathering up all the other constants to the other side of the equation so that we will have:
$(ax+b)^2 = c$ And we proceed in a similar way to our $(x+5)^2 = 7$ example.

WHAT IS COMING NEXT: The next section will build on these ideas, so make sure you understand them, and that you ALWAYS REMEMBER PLUS OR MINUS WHEN YOU TAKE THE SQUARE ROOT OF BOTH SIDES OF AN EQUATION!!!!!!!!!!!!!!!!!!!!!!!!!!