QUADRATIC FORMULA ~~(7.4)~~

HOW TO SOLVE ANY QUADRATIC EQUATION: First of all, make sure that you know the difference between *a* quadratic equation and *The Quadratic Formula*. A quadratic equation is *any* equation that is *second* degree. *The Quadratic Formula* is the *formula* used to solve any quadratic equation. You have probably heard of this formula, and some of you have probably already used it. Actually any quadratic equation *can* be solved by completing the square, but the Quadratic Formula gives us a direct way to get the solutions that is often quicker.

WHERE DOES IT COME FROM?: You are certainly NOT responsible for deriving the Quadratic Formula. I show the derivation here for those that are interested and so that you are at least aware that it is a direct consequence of Solving by Completing the Square, which in turn was dependent upon Extracting Roots. First: The equation must be set equal to 0.

1. $ax^2 + bx + c = 0$ Start with a general second degree equation equal to *zero*.
2. $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ Divide every term by a to get coefficient of x^2 to be 1, & keep equation equivalent.
3. $x^2 + \frac{b}{a}x + \_\_\_ = -\frac{c}{a} + \_\_\_$ Move constant to RIGHT & set up blanks for Completing the Square.

Now figure out what has to be added: $\left[\left(\frac{1}{2}\right)\left(\frac{b}{a}\right)\right]^2 = \left(\frac{b}{2a}\right)^2$

3. $x^2 + \frac{b}{a}x + \underline{\left(\frac{b}{2a}\right)^2} = -\frac{c}{a} + \underline{\left(\frac{b}{2a}\right)^2}$ LEFT side is now a Perfect Square Trinomial
4. $\left(x + \frac{b}{2a}\right)^2 = -\left(\frac{c}{a}\right)\left(\frac{4a}{4a}\right) + \left(\frac{b^2}{4a^2}\right)$ LEFT - Binomial Squared; RIGHT - Common denominator:
5. $\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}}$ Extract Roots after combining RIGHT to *one* fraction.
6. $x + \frac{b}{2a} = \pm\frac{\sqrt{b^2 - 4ac}}{2a}$ "Rearrange" numerator terms & simplify square root of denominator.
7. $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$ Isolate x
8. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Add fractions together, using common denominator, $2a$.

QUADRATIC FORMULA: Here it is:
IF $ax^2 + bx + c = 0$ THEN:

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Gives the solutions for x to the above equation.

REMEMBER: The equation MUST start set equal to 0 *before* using the formula!
ALSO: Notice the position of the fraction bar under the *entire* numerator. *Please* be NEAT & CAREFUL about this placement!

DO I HAVE TO MEMORIZE IT?: YES! Rather, I hope you will LEARN it from practicing. But that is the BEST way to learn it is to PRACTICE, PRACTICE, PRACTICE, $\cdots$. Furthermore, just "substituting" the correct numbers into the correct formula is only about HALF the problem. You must know how to correctly SIMPLIFY what you get.

EXAMPLE: SOLVE using the Quadratic Formula:

1. $3x^2 - 7x = 4$ FIRST the equation MUST be set equal to 0 *before* the Formula can be applied:
2. $3x^2 - 7x - 4 = 0$ NOW determine: $a = 3, \quad b = -7, \quad c = -4$. *Watch* your *signs*.
3. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ To learn quickly, WRITE and SAY the Formula each time.
4. $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-4)}}{2(3)}$ SHOW your substitution step.
5. $x = \dfrac{7 \pm \sqrt{49 + 48}}{6}$ Proceed with the Arithmetic.
6. $x = \dfrac{7 \pm \sqrt{97}}{6}$ This one required little simplification.

WARNING: Notice in Step 1, that the correct substitution for $b^2 = (-7)^2$ NOT -7^2. Remember our discussion in class as to Order of Operations: $-7^2 = -49$. If you wirte -7^2 and then 49 in the next step, you have made *two* errors!

NEXT EXAMPLE: SOLVE using the Quadratic Formula:

1. $3x^2 - 4x + 4 = 0$ IS set equal to 0 *ready* for the Formula, with $a = 3, \quad b = -4, \quad c = 4$.
2. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ WRITE and SAY the Formula.
3. $x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(4)}}{2(3)}$ SHOW your substitution step.
4. $x = \dfrac{4 \pm \sqrt{16 - 48}}{6}$ Proceed with the Arithmetic.
5. $x = \dfrac{4 \pm \sqrt{-32}}{6}$
6. $x = \dfrac{4 \pm i\sqrt{(16)(2)}}{6}$ "Take i out"
7. $x = \dfrac{4 \pm 4i\sqrt{2}}{6}$ Simplify Radical (You can combine last two steps easily.)
8. $x = \dfrac{2(2 \pm 2i\sqrt{2})}{6}$ Factor a 2 out of numerator to *cancel* with denominator:
8. $x = \dfrac{2 \pm 2i\sqrt{2}}{3}$ So you can see on a problem like this that the substitution was *just* the *beginning*!

WARNING: One of the MOST common MISTAKES is "cancelling" an "added term" from the numerator in one of the last steps. This would be a VSE - Very Serious Error - DON'T DO IT!

NOTE: I have NO problem with you combining *some* steps. I tried to put in *everything* in these examples. I will ask you to SHOW your SUBSTITUTION STEP, so that I know that you are doing this correctly, and to discern where possible errors are coming from. After that you may combine some steps, but on a problem as long as this last one, I would doubt that anyone would do all the steps in their head, so put down the steps that you need, rather than working it on scratch paper and just giving me an answer - UNacceptable!

LAST EXAMPLE: SOLVE using the Quadratic Formula:

1. $x^2 + 5x + 6 = 0$ IS set equal to 0 *ready* for the Formula, with $a = 1$, $b = 5$, $c = 6$.

2. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ WRITE and SAY the Formula.

3. $x = \dfrac{-5 \pm \sqrt{5^2 - 4(1)(6)}}{2(1)}$ SHOW your substitution step.

4. $x = \dfrac{-5 \pm \sqrt{25 - 24}}{2}$ Proceed with the Arithmetic.

5. $x = \dfrac{-5 \pm \sqrt{1}}{2}$

6. $x = \dfrac{-5 \pm 1}{2}$

Now this serves to illustrate what must be done when the radical "comes out even". It would not be proper to leave the answer in this form because the two individual answers:
$\dfrac{-5+1}{2}$ and $\dfrac{-5-1}{2}$ *simplify* to two other numbers: $\dfrac{-4}{2} = -2$ and $\dfrac{-6}{2} = -3$ respectively.

7. $x = -2$ or $x = -3$ Answers correctly simplified.

Actually, if you had just been asked to SOLVE this equation any way you wanted to, you would have factored one this simple; however, I used this to illustrate that when you do use the Quadratic Formula, or Solve by Completing the Square, or by Extracting Roots and the solution comes out with NO Radical left, you MUST simplify the two numbers individually, rather than leave it in "± notation."