RATIONAL EXPONENTS ~~6.6~~

Assumptions: I am assuming in this discussion that all numbers *implied* by the use of the fractional exponents will be REAL NUMBERS. This will become clearer in the next lesson when I discuss Complex Numbers that are NOT REAL NUMBERS and do NOT always behave as "nicely" as real numbers!

REMINDERS ON NEGATIVE EXPONENTS DO's and DON'Ts: Things that you CAN DO:

1. $A^{-n} = \dfrac{1}{A^n}$ 2. $\left(\dfrac{A}{B}\right)^{-n} = \left(\dfrac{B}{A}\right)^{n}$ 3. $\dfrac{A^nB^{-m}}{C^k} = \dfrac{A^n}{C^kB^m}$ 4. $\dfrac{A^n}{C^kB^{-m}} = \dfrac{A^nB^m}{C^k}$

The last three are just a consequence of the basic relation in #1, but keeping them in mind and being comfortable working with them will *save you* from the extra trouble of having to go through steps with complex fractions. Now for things you can NOT DO:
WARNING: A NEGATIVE EXPONENT DOES *NOT* MAKE THE NUMBER NEGATIVE, AND NEGATIVE NUMBERS HAVE NOT SUDDENLY BECOME FRACTIONS!!!!!!
These are two, unfortunately, not uncommon, but Very Serious Errors that students make.
For example: 8^{-1} does NOT equal -8 CORRECT: $8^{-1} = \dfrac{1}{8}$
Some have apparently made what I call "cancelling errors" in their work and convinced themselves that their wrong thoughts are correct, for instance, they will have these *erroneous* steps on the test:
$8^{-1} = -8 = \dfrac{1}{8}$ WRONG!!!!!!!!!! Two BAD ERRORS!
So, "Just getting the right answer" is not enough if I see that there are serious errors in getting to it!
-3 does NOT equal $\dfrac{1}{3}$. These are the *same* two numbers that they have always been... How could a *negative three* equal a *positive one − third*? Do you see how little sense this makes? However, I can't remember grading a test involving negative exponents that someone didn't make these mistakes. You be the class to start a new trend and learn the difference!

MORE FRACTIONS? - WHAT DO THEY MEAN UP *THERE*?: This may be what you are thinking when you see an expression like $x^{\frac{1}{2}}$. Our clues to the meaning of this must come from our previous knowledge of Exponent Rules (Reviewed in Notes #1). You probably saw these exponent rules for the first time years ago, and learned the basics of working with simple whole number exponents. Then some time later, perhaps for the first time in MS101, you were introduced to the idea of the negative exponent being the same thing as a reciprocal or inverse. But, hopefully, you noticed that the basic Exponent Rules DID NOT CHANGE. They had to be consisitent to include the negative integers as well as the whole numbers. The same is true now when the idea of a *rational* or *fractional* exponent is introduced .... The basic EXPONENT RULES for Multiplying together same bases, Dividing same bases, or Raising a Power to a Power DO NOT CHANGE!

TRY THINGS OUT: What would happen if we took our expression above, $x^{\frac{1}{2}}$ and squared it?
$\left(x^{\frac{1}{2}}\right)^2$ According to our "Power Rule," this must:
$= x^{(\frac{1}{2})2} = x^1 = x$
Now, ask yourself, what is it that is *squared* that gives me back x... It is $\sqrt{x}$.
$\left(\sqrt{x}\right)^2 = x$ so do you see that we can conclude that:

$x^{\frac{1}{2}} = \sqrt{x}$ Similarly, we can conclude that:
$x^{\frac{1}{3}} = \sqrt[3]{x}$ and generally
$x^{\frac{1}{n}} = \sqrt[n]{x}$ for n a positive integer.
Also to Keep the Basic Exponent Rules consistent, if we had $x^{\frac{m}{n}}$, we would notice:
$x^{\frac{m}{n}} = x^{\left(\frac{1}{n}\right)(m)} = \left(\sqrt[n]{x}\right)^m$ Also:
EQUATION #2: $x^{m/n} = \sqrt[n]{x^m}$ This is better for use in equations because it looks neater.
However, for evaluating numbers the first one is *better* because it keeps the numbers that you have to work with smaller.

EVALUATE USING EQUATION #1 $x^{m/n} = \left(\sqrt[n]{x}\right)^m$ **:** To EVALUATE means to *find the value of*, so in the first excercises and on the Worksheets, you will be getting used to this new exponent by literally finding out what the value of things like $64^{1/2}$ is. Use the above formula and proceed, but try to learn to start thinking in the language of fraction exponents ... New notation is just like a foreign language and when you learn to "think" in the new language, you don't have to go back through the "translation" process.
$64^{1/2} = \sqrt{64} = 8$ Now try:
$\left(\frac{27}{8}\right)^{-4/3} = \left(\frac{8}{27}\right)^{4/3} = \left(\sqrt[3]{\frac{8}{27}}\right)^4 = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$
You can see that you needed the previous knowledge about negative exponents "making inverses" to be able to do this problem.
WARNING: Notice that the Negative Exponent did NOT indicate taking an inverse of the exponent:
$8^{-4/3} = \frac{1}{8^{4/3}}$ CORRECT
$8^{-4/3}$ does NOT equal $8^{3/4}$ WRONG!!!!!!!!!!!

EXPONENT TO RADICAL FORM: Use Equation #2 to change an expression or equation from rational exponent form to radical form ... it *looks neater* than the Equation #1 form. What you have to be CAREFUL of here is Order of Operations. For example:
$\left(3x\right)^{4/5} = \sqrt[5]{(3x)^4} = \sqrt[5]{81x^4}$ BUT:
$3\left(x\right)^{4/5} = 3\sqrt[5]{x^4}$

WORK WITH RATIONAL EXPONENTS: In problems where you are told to express your results using positive *exponents only*, the idea is to USE THE EXPONENT RULES — NOT TO CHANGE BACK TO RADICAL FORM! On these problems, changing back to radical form will, in general, just cause trouble and not be a help.
EXAMPLES: Simplify, using exponent rules, do NOT leave any negative or zero exponent in your answer:
$\left(x^{1/2}\right)\left(x^{2/3}\right) = x^{(1/2)+(2/3)}$ NOTE: We have just applied our exponent rule that says to *add* exponents when we multiply the *same* base. Do your scratch work and add: $\frac{1}{2} + \frac{2}{3} = \frac{3}{6} + \frac{4}{6} = \frac{7}{6}$, so our Final Answer: $x^{7/6}$
The other problems are worked in similar ways, in that you apply the appropriate exponent rule(s) for multiplication, division, and powers, as needed. The only difference to problems that you have done previously is that now your *arithmetic* will involve some fractions, so just be careful! (Pay attention to the warnings!)