

SOLVING RATIONAL (FRACTION) EQUATIONS ~~(5.6A)~~

WARNING: Follow directions. Notice that these are equations to be *SOLVED*, *NOT* expressions to be *SIMPLIFIED*. Follow basic solving procedure: Each step is an equivalent equation. (See CHECK below)

Step 1: Determine the Least Common Denominator of the entire equation. This is very similar, as you can see, to simplifying rational expressions. You need to factor denominators where necessary. But, now rather than multiplying by an equivalent of "one" to get each term to have the common denominator, we use the fact that we are in an equation.

Step 2: Multiply both sides of the equation by the LCD. Remember this would have to "distribute through" each term, so the safest way is to multiply the LCD by each term on both sides. The effect is that the denominator of each fraction is "eliminated".

Step 3: Now that the equation no longer contains fractions, you need to stop and analyze: What kind of equation do you have now? Is it linear (1st degree) or quadratic (2nd degree)? What type of equation you have will determine how to proceed from here. As you can see from our general discussion of solving equations, we have used a new method to change the form of the equation back to one that we were already familiar with - either linear or quadratic.

Step 4: If the equation is linear, proceed in the normal manner to get all your x -terms (or whatever your unknown is) on one side of the equation and all your constants on the other side of the equation, finally isolating x . Alternately, if the equation is quadratic, it should be set equal to zero and factored. Remember that the only way a product can be zero is if one of the factors is zero, so that then each of the factors is set individually equal to zero.

Step 5: CHECK: This is one of those checks that is necessitated because of the "Exceptions" discussed in "Solving Equations". Now, you might wonder, what did we do that could have created extra solutions? Didn't we just multiply both sides of the equation by the same thing and proceed in a normal manner? The problem is that when we multiply by an expression containing a variable as we do in this type of problem to "clear out" the variable from the denominator, we don't know the value of the expression containing the variable. IF that expression that we multiply by has value *ZERO*, then we have broken one of our rules of maintaining equivalent equations. The way to make sure that this did *NOT* happen is simply to CHECK the answer against the factored denominators and make sure that it did *NOT* make any of these *ZERO*! I won't make you do a "formal check" of every answer; but you need to do a mental check. Obviously these *extra* solutions could not be *real* solutions since they would make the original equation undefined.

WARNING: Not using the LCD can create an equation of higher degree that can be hard to solve.

CROSS MULTIPLICATION is a method that is a consequence of multiplying a rational equation by its LCD ONLY when there is a single fraction on each side of the equation. For example, if $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$, which is the same result as multiplying both sides by the LCD, bd .

WARNING: Sometimes cross multiplication can create an equation of large degree because the product of the 2 denominators is not the LCD. Be sure to factor each and check this out.