

3.11 Related Rates

Steps for Solving Related Rates Problems

1. Make a drawing of the situation if possible.
2. Use letters to represent the variables involved in the situation – say x , y .
3. Identify all rates of change given and those to be determined. Use calculus notation $\frac{dx}{dt}$, $\frac{dy}{dt}$, etc, to represent them.
4. Determine an equation that both
 - a. Involves the variables of step 2 and
 - b. Will involve the derivative of step 3, when differentiated
(You may need some geometrical formulas to do this)
5. Differentiate (by implicit differentiation) the equation of step 4
6. Substitute all known values into the differentiated equation
7. Use algebraic manipulation, if necessary, to solve for the desired unknown rate or quantity.

Example 1

Water pours into a fish tank at a rate of $3 \text{ ft}^3 / \text{min}$. How fast is the water level rising if the base of the tank is a rectangle of dimensions $2 \times 3 \text{ ft}$?

Example 2

A spherical snowball melts in such a way that the instant at which its radius is 20 cm , its radius is decreasing at 3 cm/min . At what rate is the volume of the ball of snow changing at that instant?

Example 3

A ladder 25 feet long is leaning against the wall of the house, while the base of the ladder is pulled away from the wall at a rate of 3 ft/sec .

- a) How fast is the top moving down the wall when the base of the ladder is 20 feet away from the wall?

- b) How fast is the angle between the ladder and the wall changing when the base of the ladder is 15 feet away from the wall?

Example 4

- (a) Water runs into a inverted conical tank at the rate of $7 \text{ ft}^3 / \text{min}$. The radius of the water's surface is always half the height of the water. How fast is the water level rising when the water is 2 feet deep?
- (b) Suppose water is leaking out of the tank at a rate of 2 cubic feet per minute in (a). How fast is the radius of surface of the water increasing when the water is 2 feet deep?

Example 5 (optional)

A kite is flying 150 m high, where the wind causes it to move horizontally at the rate of 5 m per second. In order to maintain the kite at a height of 150 m, the person must allow more string to be let out. At what rate is the string being let out when the length of the string already out is 250 m?

Example 6 (optional)

An airplane, flying at 450 km/hr at a constant altitude of 5 km, is approaching a camera mounted on the ground. Let θ be the angle of elevation above the ground at which the camera is pointed.

When $\theta = \frac{\pi}{3}$, how fast does the camera have to rotate in order to keep the plane in view?

HOMEWORK: 1, 3, 5, 7, 9, 11, 15, 17, 19, 21, (27, 29)