

ALGEBRA II WITH TRIGONOMETRY EXAMINATION
2002

ALABAMA STATE-WIDE MATHEMATICS CONTEST

Construction of this test directed
by
Susan P. Slattery
Alabama State University

INSTRUCTIONS

This test consists of 50 multiple choice questions. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a “perfect paper” with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.) Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

The questions have not been arranged in order of difficulty.

All variables and constants represent real numbers, except when a particular problem indicates otherwise.

Diagrams are not necessarily to scale.

Editing by Jeff Dodd, Tom Leathrum, Steve White, Jason Huffman, and Jan Case
Typesetting by Tom Leathrum and Jeff Dodd
Dept. of Mathematical, Computing and Information Sciences
Jacksonville State University

1. Simplify $\frac{\frac{3}{2} - \frac{2}{3}}{\frac{2}{3} + \frac{3}{2}}$.
 (A) $\frac{5}{13}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{3}{5}$ (E) $\frac{5}{6}$
2. The value of $a - b^{b-a}$ when $a = 1$ and $b = -2$ is:
 (A) $\frac{7}{8}$ (B) 1 (C) -1 (D) $\frac{9}{8}$ (E) $\frac{1}{27}$
3. The fraction $\frac{3(x-1)}{2x^2+x-1}$ was obtained by adding the fractions $\frac{A}{x+1}$ and $\frac{B}{2x-1}$.
 What is $A + B$?
 (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
4. A large plane makes the trip from Los Angeles to Dallas in 3 hours. A small plane makes the same trip in 3.5 hours. If both planes leave Los Angeles at the same time, how long they will have been flying, to the nearest tenth of an hour, when the small plane is twice as far from Dallas as the large plane?
 (A) 1.4 hours (B) 1.9 hours (C) 2.2 hours
 (D) 2.6 hours (E) The situation never happens.
5. How many real roots does the polynomial $p(x) = 2x^3 + 5x^2 + 9x$ have?
 (A) zero (B) one (C) two (D) three (E) four
6. Find the perimeter of a right triangle in which one angle measures 60° , while the hypotenuse has a length of 4 cm.
 (A) $10 + \sqrt{2}$ cm (B) 8 cm (C) $6 + 2\sqrt{3}$ cm (D) $4 + \sqrt{6}$ cm (E) $4\sqrt{2}$ cm
7. Find the distance between the points of intersection of the graphs of $y = 2x^2 - 4x + 1$ and $2x - y = -21$.
 (A) $\sqrt{51}$ (B) $7\sqrt{5}$ (C) $\sqrt{21}$ (D) $3\sqrt{7}$ (E) $9\sqrt{11}$
8. Find an equation for the perpendicular bisector of the line segment connecting the points $(-2, 5)$ and $(3, -7)$.
 (A) $10x - 24y = 29$ (B) $24x + 10y = -5$ (C) $10x + 24y = -19$
 (D) $24x - 10y = 15$ (E) $24y - 10x = -1$
9. Find the y -intercepts of the circle having center $(4, 5)$ and radius 5.
 (A) $(0, -2)$ and $(0, 3)$ (B) $(0, 4)$ and $(0, -5)$ (C) $(0, 1)$ and $(0, 6)$
 (D) $(0, 8)$ and $(0, 2)$ (E) $(0, 5)$ and $(0, 0)$

10. The solution set of the inequality $|6x - 5| \geq 10$ is:

- (A) $(-\infty, -\frac{5}{6}]$ (B) $(-\infty, -\frac{5}{2}] \cup [\frac{5}{6}, \infty)$ (C) $[\frac{5}{2}, \infty)$
(D) $[-\frac{5}{6}, \frac{5}{2}]$ (E) $(-\infty, -\frac{5}{6}] \cup [\frac{5}{2}, \infty)$

11. The domain of the function $f(x) = \sqrt{18 - 2x^2}$ is $[a, b]$ where $b - a =$

- (A) 0 (B) 3 (C) 6 (D) 9 (E) 18

12. Among all the solutions, real and complex, of the following system of equations:

$$\begin{aligned}x^2 + y^2 + x + y &= 8 \\xy + x + y &= 5\end{aligned}$$

the least possible real part of x is:

- (A) 0 (B) -1 (C) -2 (D) -3 (E) -4

13. The graph of $y = -x^2 - 6x - 3$ has:

- (A) its lowest point at $(0, 3)$ (B) its highest point at $(-3, 24)$
(C) its lowest point at $(-3, -24)$ (D) its highest point at $(-3, 6)$
(E) its lowest point at $(-3, 0)$

14. The graph of $9x^2 - 4y^2 - 54x - 16y + 29 = 0$ is symmetric with respect to which lines?

- (A) $x = -2$ and $y = 3$ (B) $x = 2$ and $y = -3$ (C) $x = 3$ and $y = -2$
(D) $x = -3$ and $y = 2$ (E) None of the above.

15. Find the smallest whole number B so that 61 can be expressed in base B using only two digits.

- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10

16. Explain how you could use the graph of $y = f(x)$ to sketch the graph of $y = -f(x + 1) + 2$.

- (A) Shift left 1 unit, reflect across the x -axis, then shift up 2 units.
(B) Shift right 1 unit, reflect across the x -axis, then shift up 2 units.
(C) Shift left 1 unit, reflect across the y -axis, then shift up 2 units.
(D) Reflect across the x -axis, shift left 1 unit, then shift down 2 units.
(E) Reflect across the y -axis, shift right 1 unit, then shift up 2 units.

17. A club has three different committees each of which needs a chairperson. How many ways can the chairpeople be selected if only 10 people are eligible for these positions, and no one can chair more than two of the committees?

- (A) 510 (B) 640 (C) 720 (D) 850 (E) 990

18. The x -axis, the y -axis, and the line through the point $(1, 3)$ having slope -3 form a triangle with area:

- (A) 2 (B) 4 (C) 6 (D) 8 (E) 10

19. Two cards are dealt from a standard deck of 52 cards. Find the probability that both cards will have the same color.

- (A) $\frac{15}{26}$ (B) $\frac{25}{51}$ (C) $\frac{7}{13}$ (D) $\frac{1}{2}$ (E) $\frac{9}{17}$

20. The solutions of the equation $\sin 2t = \sin t$ in the interval $0 \leq t < 2\pi$ are:

- (A) $0, \pi$ (B) $0, 2\pi/3, 4\pi/3, 3\pi/2$ (C) $0, \pi/6, 5\pi/6$
(D) $0, \pi/3, \pi, 5\pi/3$ (E) $0, 5\pi/6, \pi, 7\pi/6$

21. Compute $(3 - 2i)^3$.

- (A) $27 - 8i$ (B) $-9 - 46i$ (C) $39 - 26i$ (D) $-35 + i$ (E) $-i$

22. If the repeating decimal $0.26767676767\dots$ is represented as a ratio of positive integers in lowest terms, then the numerator of this ratio is:

- (A) 168 (B) 17 (C) 267 (D) 53 (E) 99

23. If $\log_6 x = a$, $\log_6 y = b$, and $\log_6 z = c$, express $6 \log_6 \left(\frac{36x^4\sqrt{z}}{y^3} \right)$ in terms of a , b , and c .

- (A) abc (B) $4a + 3b + \frac{1}{2}c + 36$ (C) $36a^4 - 6b^3 + 3\sqrt{c}$
(D) $\frac{216a^4\sqrt{c}}{b^3}$ (E) $24a - 18b + 3c + 12$

24. A store is having a 30% off sale. A CD player is on sale for \$203. What would the same CD player sell for if it were on sale for only 20% off?

- (A) \$61 (B) \$232 (C) \$345 (D) \$223 (E) \$142

25. From 100 pounds of pure wheat flour, a certain amount was taken and replaced by corn meal, and the resulting combination was mixed uniformly. Later, from this preliminary mixture, the same amount was again taken and replaced by corn meal, and the resulting combination mixed uniformly. The final mixture was 16 parts wheat to 9 parts corn. The ratio of the amount of corn to the amount of wheat in the preliminary mixture can be expressed as a ratio of positive integers in lowest terms, and the numerator and denominator of the resulting fraction sum to:

- (A) 9 (B) 6 (C) 10 (D) 5 (E) 12

26. The intensity of a light source varies inversely as the square of the distance from the source. If the distance from a source increases by 25%, by what percent does the intensity decrease?

- (A) 6.25% (B) 20% (C) 36% (D) 50% (E) 64%

27. The area of a rectangle is 100 square cm. If a square is built using a diagonal of this rectangle as one of its sides, the square has area 700 square cm. What is the perimeter of the rectangle?

- (A) 30 cm (B) 45 cm (C) 50 cm (D) 60 cm (E) 90 cm

28. The complex conjugate of $\frac{6-3i}{4+3i}$ is:

- (A) $\frac{6+3i}{4-3i}$ (B) $\frac{6+3i}{4+3i}$ (C) $\frac{6-3i}{4-3i}$ (D) $\frac{3-6i}{4+3i}$ (E) $\frac{6-3i}{3+4i}$

29. The expression $\frac{\tan \alpha}{1+\sec \alpha} + \frac{1+\sec \alpha}{\tan \alpha}$ can be simplified to the expression:

- (A) 1 (B) $2 \csc \alpha$ (C) $2 \sec \alpha - \tan \alpha$ (D) $\cot \alpha$ (E) $4 \sin \alpha$

30. A cylindrical tube is to be made by joining opposite ends of a very thin rectangular sheet of plastic. Find the height of the tube given that the sheet of plastic has an area of 200 in^2 and the tubing has a volume of 200 in^3 .

- (A) π^2 in (B) $50/\pi$ in (C) 5π in (D) 25 in (E) $3\sqrt{5\pi}$ in

31. Find the sum of the first 50 terms of the arithmetic sequence $\{a_n\}_{n=1}^{\infty}$ for which $a_5 = -15$ and $a_{15} = 35$.

- (A) 4035 (B) 4165 (C) 4225 (D) 4375 (E) 4645

32. If the product $(x+y)^8$ is fully expanded and like terms collected, the coefficient of y^3x^5 in the resulting expression is:

- (A) 128 (B) 70 (C) 28 (D) 96 (E) 56

33. How many different arrangements are there of the letters in the word BANANA?

- (A) 180 (B) 72 (C) 60 (D) 120 (E) 480

34. A river runs at a constant speed relative to the shore and a boat travels on the river at a constant speed relative to the river. The boat's speed on a trip upstream is 6 miles per hour and its speed on the return trip downstream is 9 miles per hour. What is the boat's average speed on the round trip in miles per hour?

- (A) $29/4$ (B) $36/5$ (C) $15/2$ (D) $22/3$ (E) $54/7$

35. The legs of a right triangle measure 5 and 12. Find the cosine of the smallest angle in the triangle.

- (A) $5/13$ (B) $5/12$ (C) $12/13$ (D) $12/5$ (E) $13/5$

36. Evaluate

$$\log_4 \sqrt[5]{32 \sqrt[3]{16 \sqrt{64 \cdot 8^{-3/4}}}}$$

(A) $\frac{83}{145}$ (B) $\frac{95}{184}$ (C) $\frac{165}{242}$ (D) $\frac{167}{240}$ (E) $\frac{121}{186}$

37. Calculate $x + y + z$ given that

$$\begin{cases} 2x + y + 3z = 12 \\ 3x - 2y - 5z = 7 \\ x + 3y + 4z = 1. \end{cases}$$

(A) 2 (B) -3 (C) 0 (D) 4 (E) -5

38. Suppose $f(x) = \begin{cases} x^2 + x & \text{if } x > 2 \\ 2x & \text{if } x \leq 2 \end{cases}$ and $g(x) = \begin{cases} x + 7 & \text{if } x \leq 0 \\ x^2 & \text{if } x > 0. \end{cases}$

How many solutions are there of the equation $f(x) = g(x)$?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

39. Solve for x : $\log_3(5x - 4) = 2 + \log_3(6 - 7x)$.

- (A) $29/34$ (B) 1 (C) $19/12$ (D) 3 (E) There are no solutions.

40. Suppose $f(x) = \sqrt[3]{\frac{2x-1}{x}}$. Calculate $f^{-1}(-1)$.

- (A) -1 (B) $1/3$ (C) $\sqrt[3]{3}$ (D) 3 (E) $\sqrt[3]{2}$

41. The ratio of sweetened lemon juice to water in lemonade is 2:7. How much lemonade can you make if you use 5 cups of sweetened lemon juice?

- (A) 7 cups (B) 17.5 cups (C) 22.5 cups (D) 24 cups (E) 30.5 cups

42. Determine the largest possible integer n such that $942!$ is divisible by 15^n .

- (A) 62 (B) 125 (C) 233 (D) 314 (E) 471

43. If a function f satisfies the equation $f(a + b) = f(a) + f(b)$ for every pair of real numbers a and b , then what is (are) the possible value(s) of $f(0)$?

- (A) Any real number. (B) Any positive real number. (C) 0 and 1 only
(D) 1 only (E) 0 only

44. If $\sin \alpha = \frac{2}{3}$ and α is an obtuse angle, then $\cos \alpha = ?$

- (A) $\frac{2}{3}$ (B) $-\frac{\sqrt{5}}{3}$ (C) $-\frac{3}{4}$ (D) $\frac{\sqrt{13}}{3}$ (E) $-\frac{\sqrt{2}}{3}$

45. One light blinks every 120 seconds and another blinks every 450 seconds. If you start timing them when they blink simultaneously, how long will it be between this first simultaneous blink and the fourth simultaneous blink?

- (A) 900 seconds (B) 1800 seconds (C) 3600 seconds (D) 5400 seconds
(E) 10800 seconds

46. Suppose that the equation $3bx^2 + 6bx = ax + 2a$ has two solutions for x and that the sum and the product of these solutions are equal. Then $b/a =$

- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) 1

47. There are unique rational numbers a , b , and c such that

$$\frac{1}{\sqrt{2} + \sqrt{3} + \sqrt{5}} = a\sqrt{2} + b\sqrt{3} + c\sqrt{30}$$

and $a + b + c =$

- (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{10}$ (D) $\frac{2}{3}$ (E) $\frac{5}{2}$

48. How many odd four digit numbers have four different digits?

- (A) 2040 (B) 2160 (C) 2240 (D) 2520 (E) 2880

49. Suppose that the system of equations $x^2 - y = a$ and $y - x = b$ has exactly one solution for x . Then what is this value for x ?

- (A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) 1 (D) -1 (E) $\frac{3}{2}$

50. How many ordered pairs of integers (x, y) satisfy

$$y^2 - xy + x = 0?$$

- (A) 1 (B) 2 (C) 3 (D) 4 (E) Infinitely many.