

ALGEBRA II WITH TRIGONOMETRY EXAMINATION
2003

ALABAMA STATE-WIDE MATHEMATICS CONTEST

Construction of this test directed
by
Susan Slattery, Alabama State University

INSTRUCTIONS

This test consists of 50 multiple choice questions. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a “perfect paper” with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.) Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

The questions have not been arranged in order of difficulty.

All variables and constants represent real numbers, except when a particular problem indicates otherwise.

Diagrams are not necessarily to scale.

Editing by Prof. Jeff Dodd, Prof. Tom Leathrum, and Prof. Steve White
Typesetting by Prof. Tom Leathrum and Prof. Jeff Dodd
Dept. of Mathematics, Computing, and Information Sciences
Jacksonville State University

1. Express the following in scientific notation: $\frac{(2.3 \times 10^4) \times (1.1 \times 10^5) \times (8 \times 10^{-3})}{(2 \times 10^2) \times (5 \times 10^{-6})}$.
- (A) 2.024×10^5 (B) 5.06×10^6 (C) 2.024×10^{10} (D) 5.06×10^{11}
(E) None of these.
2. $\left(\sqrt[6]{\sqrt[3]{10}}\right)^9 =$
- (A) $\frac{1}{10}$ (B) $\frac{\sqrt{10}}{10}$ (C) $\sqrt{10}$ (D) 10 (E) $10\sqrt{10}$
3. How many solutions does the equation $|x + 2| = 2x$ have?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) Infinitely many.
4. $(x + 1) + (x - 2) + (x + 3) + (x - 4) + \cdots + (x + 99) + (x - 100) =$
- (A) $100x - 1$ (B) $100x - 25$ (C) $100x - 50$ (D) $100x - 100$
(E) None of these.
5. The squares of two consecutive integers differ by 1987. What is the product of the two integers?
- (A) 993 (B) 987,042 (C) 82,141 (D) 1,653 (E) 59,210
6. Two executives from the same office are driving separately to a conference. The first leaves at 1:00pm, traveling at 50 mph. The second leaves at 2:00pm, traveling at 70 mph. Assuming they both travel non-stop along the same route, at what time will the second executive pass the first?
- (A) 3:00pm (B) 3:30pm (C) 4:00pm (D) 4:30pm (E) 5:00pm
7. The scale on a map states that 1cm represents 6 miles. How many square miles would be represented by an area on the map of 240cm^2 ?
- (A) 1,440 (B) 57,600 (C) 600 (D) 40 (E) 8,640
8. Find the distance from the point $(5, 6)$ to the line $x + y = 3$.
- (A) $5\sqrt{3}$ (B) 6 (C) $2\sqrt{7}$ (D) 3 (E) $4\sqrt{2}$

9. Suppose that for any integer n , $f(n) = \begin{cases} n + 3 & \text{if } n \text{ is odd} \\ \frac{n}{2} & \text{if } n \text{ is even.} \end{cases}$
If k is odd and $f(f(f(k))) = 27$, find the sum of the digits in k .
- (A) 6 (B) 5 (C) 4 (D) 3 (E) 2
10. Solve for n : $4^n + 4^n + 4^n + 4^n = 4^{16}$.
- (A) 2 (B) 9 (C) 4 (D) 15 (E) No solution.
11. Simplify the expression: $\frac{1 + \frac{1}{1+b}}{1 - \frac{1}{1+b}}$.
- (A) -1 (B) $\frac{2+b}{b}$ (C) $\frac{2b}{1+b}$ (D) $\frac{1+b}{1-b}$ (E) 1
12. For how many real numbers k does the equation $9x^2 + kx + 4 = 0$ have exactly one solution?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
13. One solution of $km^2 + 10m = 8$ is $m = -4$. Find the other solution (for m).
- (A) $\frac{2}{3}$ (B) 3 (C) $-\frac{1}{2}$ (D) -1 (E) 4
14. Given points $A = (-2, 5)$ and $B = (6, -1)$, find the point C from the list below so that A , B , and C are colinear.
- (A) $C = (10, -4)$ (B) $C = (2, -2)$ (C) $C = (4, 3)$ (D) $C = (-6, 9)$
(E) $C = (0, 0)$
15. Which one of the following functions is an even function?
- (A) $y = \csc x$ (B) $y = \tan x$ (C) $y = \cos x$ (D) $y = \sin x$ (E) None of these.
16. The distance a body falls from rest varies directly as the square of the time it falls. If an object falls 1024 feet in 8 seconds, how far will it fall in 12 seconds?
- (A) 1536 feet (B) 2304 feet (C) 3072 feet (D) 8126 feet (E) 12288 feet

17. A line has x -intercept $(5, 0)$ and is perpendicular to the line $2x + 8y = 10$. Find the y -intercept of the line.

- (A) $(0, 10)$ (B) $(0, -8)$ (C) $(0, 16)$ (D) $(0, -20)$ (E) $(0, -2)$

18. Find the coefficient of the term containing x^2 in the expansion of $\left(2x^3 - \frac{1}{x}\right)^{10}$.

- (A) -960 (B) 8 (C) -240 (D) -30 (E) 520

19. Which of the following is equivalent to $\frac{6 + 2i}{5 - 4i}$, where $i = \sqrt{-1}$?

- (A) $\frac{6}{5} - \frac{1}{2}i$ (B) $\frac{3}{2} + \frac{2}{5}i$ (C) $\frac{5 + 4i}{6 - 2i}$ (D) $\frac{38}{9} - \frac{34}{9}i$ (E) $\frac{22}{41} + \frac{34}{41}i$

20. If x , y , and z solve the system of equations

$$2x + 3y - z = -8$$

$$5x - y - z = 3$$

$$x - 6y - 3z = 1$$

then what is $x + y + z$?

- (A) -4 (B) 5 (C) -7 (D) 3 (E) 0

21. Steve can mow his entire lawn in 50 minutes less time with his power mower than he can with his push mower. One day the power mower broke after he had been mowing for 30 minutes. He finished the lawn with the push mower in 20 minutes. How many minutes does it take Steve to mow his entire lawn with the power mower?

- (A) $12\sqrt{10}$ (B) $10\sqrt{15}$ (C) $20\sqrt{3}$ (D) $16\sqrt{5}$ (E) $30\sqrt{2}$

22. Find the domain of the function $f(x) = \sqrt{6 - x - x^2}$.

- (A) $(-\infty, 6]$ (B) $[-2, 3]$ (C) $[3, \infty)$ (D) $[-3, 2]$ (E) $[-6, 6]$

23. Which is **not** a factor of $a^8 - b^8$?

- (A) $a - b$ (B) $a + b$ (C) $a^2 + b^2$ (D) $a^3 + b^3$ (E) $a^4 + b^4$

24. Solve for x : $ix^2 - 4x + i = 0$, where $i = \sqrt{-1}$.

- (A) $(-2 \pm \sqrt{5})i$ (B) $2 - i$ (C) $-4 \pm 2i$ (D) $2 \pm \sqrt{2}$
(E) No real or complex solutions.

25. The equation $\sqrt{x} - 1 = \sqrt{x - 2}$ has
(A) One solution. (B) Two solutions whose sum is greater than 6.
(C) Two solutions whose sum is less than 4.
(D) Two solutions both lying between 2 and 3. (E) No solutions.
26. Given $f\left(\frac{x+1}{x-1}\right) = 2x$, what is $f(3)$?
(A) 2 (B) 3 (C) 4 (D) 5 (E) 6
27. The points $A = (-1, 2)$ and $B = (3, -6)$ lie on opposite ends of a diameter of a circle. Where does the circle intersect the x -axis?
(A) $(1, 0), (-2, 0)$ (B) $(4, 0), (-4, 0)$ (C) $(3, 0), (-6, 0)$ (D) $(5, 0), (-3, 0)$
(E) The circle doesn't intersect the x -axis.
28. Solve $2^{16^x} = 16^{2^x}$ for x .
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$ (E) $\frac{5}{6}$
29. If $i = \sqrt{-1}$, what is i^{2003} ?
(A) 0 (B) 1 (C) -1 (D) i (E) $-i$
30. Find the sum of the solutions of the equation $\log_2 x + \log_2(x + 6) = 4$.
(A) 6 (B) -6 (C) 2 (D) -2 (E) None of these.
31. The sum of 20 consecutive integers is 150. What is the smallest integer in the list?
(A) 1 (B) 7 (C) -2 (D) -10 (E) 0
32. How many different "words" can be made by rearranging the letters in the word TOMATO? (The "words" do not have to make sense.)
(A) 10 (B) 30 (C) 90 (D) 180 (E) 360
33. If $x - y = 3$ and $x^2 + y^2 = 10$, what is xy ?
(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{3}{4}$ (D) $\frac{4}{5}$ (E) $\frac{5}{6}$
34. If the point (x, y) is one-third of the way from $(-2, 6)$ to $(6, -8)$, then what is $x + y$?
(A) 0 (B) $\frac{4}{3}$ (C) $-\frac{2}{3}$ (D) -1 (E) 2

35. Which one of the following is equivalent to $2^{\log_3 5}$?
- (A) $2^{\log_5 3}$ (B) $5^{\log_3 2}$ (C) $3^{\log_2 5}$ (D) $5^{\log_2 3}$ (E) $3^{\log_5 2}$
36. If you have one penny, one nickel, one dime, one quarter, and one half-dollar, how many different amounts of money can be made using one or more of these coins?
- (A) 31 (B) 5 (C) 15 (D) 24 (E) 9
37. Given that the sum of two non-zero real numbers is 11, then the possible values of the sum of the reciprocals of these numbers include
- (A) all positive real numbers, but not all negative real numbers.
 (B) all negative real numbers, but not all positive real numbers.
 (C) all non-zero x such that $|x| < 11$, but not all x such that $|x| \geq 11$.
 (D) all non-zero real numbers. (E) None of the above.
38. At a school dance with 120 students, $1/6$ of the girls chose $1/4$ of the boys to be their partners for a slow song. How many boys were not asked to dance for that song?
- (A) 12 (B) 24 (C) 36 (D) 48 (E) 60
39. Find $\sum_{k=1}^{99} \frac{1}{\sqrt{k+1} + \sqrt{k}}$.
- (A) 10 (B) 9 (C) 8 (D) 7 (E) 6
40. Suppose that $a_{n+1} = 3a_n - 20$ and $a_1 = 9$, find a_7 .
- (A) 547 (B) 28 (C) 0 (D) -325 (E) -719
41. If $\sin^2 x - 3 \sin x \cos x - 4 \cos^2 x = 0$ and $0 < x < 90^\circ$ then what is $\tan x$?
- (A) 1 (B) 4 (C) $\sqrt{3}$ (D) 2 (E) None of these.
42. Find an equation for the ellipse with x -intercepts of $(-4, 0)$ and $(4, 0)$ and foci at the points $(-2, 0)$ and $(2, 0)$.
- (A) $16x^2 + 4y^2 = 256$ (B) $12x^2 + 16y^2 = 192$ (C) $8x^2 + 6y^2 = 128$
 (D) $2x^2 + 6y^2 = 32$ (E) $18x^2 + 8y^2 = 288$

43. Describe how you could obtain the graph of $y = 2f(x-2)$ from the graph of $y = f(x)$.

(A) Shift the graph of f down 2 units, then to the left 2 units.

(B) Shift the graph of f up 2 units, then to the left 2 units.

(C) Shift the graph of f to the right 2 units, then multiply all the y -coordinates by $\frac{1}{2}$.

(D) Shift the graph of f to the left two units, then multiply all the y -coordinates by 2.

(E) None of these.

44. Suppose that θ is an angle that terminates in the second quadrant such that $\sec \theta = -3$. Find $\tan \theta$.

(A) -4 (B) $-\frac{1}{\sqrt{3}}$ (C) 5 (D) $-2\sqrt{2}$ (E) Undefined.

45. How many solutions does the equation $|x + 4| + |x - 2| = 6$ have?

(A) 0 (B) 1 (C) 2 (D) 3 (E) Infinitely many.

46. How many integers does the solution set of $\frac{x-3}{x+1} \leq 0$ contain?

(A) 1 (B) 2 (C) 4 (D) 6 (E) 8

47. How many functions can be defined from a domain $D = \{1, 2, 3\}$ onto a range $R = \{4, 5\}$?

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

48. Find the sum: $\frac{3}{4} + \frac{3}{8} + \frac{3}{16} + \dots$

(A) 1 (B) 1.5 (C) 2.25 (D) 3.75 (E) 4

49. Centerville City Council is composed of 5 liberals and 4 conservatives. Four members are to be chosen at random as delegates to a convention. In how many ways can exactly 2 liberals and 2 conservatives be chosen?

(A) 4 (B) 6 (C) 15 (D) 35 (E) 60

50. If I made a list of all the 3-digit numbers whose digits are 1, 2, 3, 4, or 5 with no repetitions allowed, and then summed all the digits of these numbers, what would this sum be?

(A) 270 (B) 450 (C) 540 (D) 630 (E) 720