

20
06

ALABAMA

STATEWIDE MATHEMATICS CONTEST



First Round : February 18, 2006
 Second Round: April 22, 2006 at The University of Alabama

ALGEBRA II WITH TRIGONOMETRY EXAM

Construction of this test directed
 by
 Robert Moore, The University of Alabama

INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a “perfect paper” with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- | | |
|---|---|
| <ul style="list-style-type: none"> • If A and B are points, then:
 \overline{AB} is the segment between A and B
 \overleftrightarrow{AB} is the line containing A and B
 \overrightarrow{AB} is the ray from A through B
 AB is the distance between A and B | <ul style="list-style-type: none"> • If A is an angle, then:
 $m \angle A$ is the measure of angle A in degrees • If A and B are points on a circle, then:
 \widehat{AB} is the arc between A and B
 $m \widehat{AB}$ is the measure of \widehat{AB} in degrees |
|---|---|

Editing by Zhijian Wu, The University of Alabama
 Printing by The University of Alabama

1. If $\sin \alpha = \frac{-5}{13}$ and α is in Quadrant III, what is the value of $\cos\left(\frac{\alpha}{2}\right)$?
- (A) $-\frac{6}{13}$ (B) $\frac{2\sqrt{3}}{13}$ (C) $-\frac{\sqrt{5}}{12}$ (D) $\frac{4\sqrt{3}}{12}$ (E) $-\frac{\sqrt{26}}{26}$
2. The degree of a polynomial $p(x)$ is no larger than 3. We know that $p(0) = 3$, $p(-x) = p(x)$, and $p(1/x) = p(x)/x^2$. Find $p(1)$.
- (A) 2 (B) 4 (C) $\boxed{6}$ (D) 3 (E) 5
3. M is a perfect square of an integer. The next larger perfect square is $M + 461$. What is M ?
- (A) $\boxed{52,900}$ (B) 57,600 (C) 48,400 (D) 62,500 (E) 44,100
4. $\sqrt{x^2 + 165} - \sqrt{x^2 - 52} = 7$. If x is positive, find x .
- (A) 15 (B) $\boxed{14}$ (C) 13 (D) 12 (E) 11
5. $\frac{x-12}{x^2+3x} - \frac{2}{x} = \frac{5}{x+3}$. Find x .
- (A) -1 (B) $\boxed{\text{No solution}}$ (C) -3 (D) -2 (E) 0
6. Suppose that m people can do a task in d days. Assuming that they all work at the same rate, how many days will it take for $m+r$ people to do the task?
- (A) $\boxed{\frac{dm}{m+r}}$ (B) $d - \frac{r}{m}$ (C) $\frac{d(m-r)}{m}$ (D) $\frac{dr}{m+r}$ (E) None of these
7. What is the smallest value assumed by the polynomial $p(x) = 4x^2 - 3x + 8$?
- (A) $\boxed{\frac{119}{16}}$ (B) $\frac{3}{8}$ (C) 8 (D) 9 (E) $\frac{15}{2}$
8. If r and s are the (complex) roots of the equation $x^2 - \sqrt{27}x + 13 = 0$, what is the value of $r^2 + s^2$?
- (A) $\sqrt{27}$ (B) $\boxed{1}$ (C) 13 (D) 53 (E) 27
9. If $f(x) = \frac{x-1}{x-2}$, what is $f^{-1}(4)$ (here f^{-1} is the inverse function of f)?
- (A) 3 (B) -3 (C) $3/2$ (D) $2/3$ (E) $\boxed{7/3}$
10. Suppose $c \neq 0$ and the roots of $3x^2 + 7x + c = 0$ are x_1 and x_2 . In simplest form in terms of c , what is the value of $\frac{1}{x_1} + \frac{1}{x_2}$?
- (A) $\frac{3}{c}$ (B) $-\frac{3}{c}$ (C) $\frac{7}{c}$ (D) $\boxed{\frac{-7}{c}}$ (E) $\frac{7}{3c}$

11. If m and n are required to be integers, how many solutions (m, n) are there to the pair of conditions $5n - 3m = 15$ and $n^2 + m^2 \leq 16$?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4
12. If $c = \log_x b$ and $d = \log_{x^2}(b^2)$, find d in terms of c .
- (A) $2c$ (B) c^{-1} (C) c (D) c^2 (E) $c/2$
13. If the arithmetic mean of two positive numbers is A and the geometric mean is B , what are the two numbers?
- (A) $A \pm \sqrt{A^2 - B^2}$ (B) $B \pm \sqrt{A^2 - B^2}$ (C) $A \pm \sqrt{A - B}$ (D) $A \pm B$ (E) $A \pm B^{-1}$
14. Suppose x and y are two unequal numbers. Their sum, difference, and product are denoted by S , D , and P , respectively. We know that $\frac{S}{D} = 7$ and $\frac{P}{D} = 24$. What is the value of P ?
- (A) 42 (B) 31 (C) 28 (D) 48 (E) 14
15. If one and one-half teams of men require one and one-half days to shovel one and one-half truckloads of sand, how long will it take one team to shovel one truckload?
- (A) 3 days (B) 2 days (C) $2/3$ days (D) 1 day (E) 1 and $1/2$ days
16. In the expansion of $(x - \sqrt{x} - 1)^7$, what is the coefficient of $x^{5/2}$?
- (A) 231 (B) 224 (C) 14 (D) 266 (E) 56
17. What is the smallest positive integer n for which $45n$ is a perfect cube of an integer?
- (A) 75 (B) 2025 (C) 625 (D) 55 (E) 245
18. Three positive numbers a , b , and c form an arithmetic progression. Oddly enough, if you increase a by 1 you get a geometric progression. Even more oddly, if you increase c by 2 you also get a geometric progression. What is the value of b ?
- (A) 8 (B) 9 (C) 10 (D) 12 (E) 18
19. Simplify $\frac{\sin x}{1 - \cos x} - \frac{\sin x}{1 + \cos x}$.
- (A) $2 \tan x$ (B) $\cos x - \sin x$ (C) $2 \cot x$ (D) $\cos x + \sin x$ (E) None of these
20. What is the value of $81^{-(2^{-2})}$?
- (A) $1/9$ (B) 43046721 (C) $1/3$ (D) 6561 (E) $\sqrt{3}/3$
21. Write the repeating decimal $0.35353535\dots$ as a fraction.
- (A) $\frac{353}{909}$ (B) $\frac{7}{18}$ (C) $\frac{353}{990}$ (D) $\frac{353}{999}$ (E) $\frac{35}{99}$

22. If $\log_{10} 2 = a$, write $\log_5 2$ in terms of a .
- (A) $\frac{a}{1-a}$ (B) $\frac{a}{1+a}$ (C) $\frac{1+a}{1-a}$ (D) $\frac{a-1}{a}$ (E) $\frac{a}{a-1}$
23. The vertex of the parabola $y = 2x^2 - 6x + c$ lies on the x -axis. What is the value of c ?
- (A) 3 (B) 0 (C) $\frac{9}{2}$ (D) 2 (E) $\frac{7}{2}$
24. The symbol $(37)_b$ represents a two-digit number written in base b . For instance, $(37)_8 = 31$, where the last number is written in base 10. Suppose that $(37)_b$ is exactly one-half of $(73)_b$. What is b ?
- (A) $\frac{11}{2}$ (B) 10 (C) 9 (D) 8 (E) 12
25. For $x \geq 0$, what is the smallest possible value of the expression $\log(x^3 - 4x^2 + x + 26) - \log(x + 2)$?
- (A) $\log 3$ (B) $\log 2$ (C) $\log 5$ (D) $\log 4$ (E) $\log 6$
26. A wholesaler buys a load of small refrigerators for 256 dollars each. He marks the refrigerators up by 25% and sells them to a merchant. The merchant adds a further markup of 25% and marks this price on the refrigerators. The items are not moving at this price, so he puts up a sign that says "25% off marked price." If a customer buys a refrigerator, how much will she pay (neglecting taxes)?
- (A) \$296 (B) \$240 (C) \$300 (D) \$320 (E) \$288
27. If the number $15!$ is written out in base ten, how many zeroes will there be at the end?
- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5
28. Suppose a and b are fixed real numbers, each greater than 1. How many solutions x are there to the equation $(\log_a x) \cdot (\log_b x) = 7$.
- (A) 1 (B) 0 (C) 4 (D) 2 (E) 3
29. The sum of the first five terms of a geometric series is 189, the sum of the first six terms is 381, and the sum of the first seven terms is 765. What is the common ratio in this series?
- (A) $\sqrt{2}$ (B) 2 (C) $2\sqrt{2}$ (D) 4 (E) $\sqrt{4}$
30. Let $z = 6 + 7i$, where $i^2 = -1$. If $1/z = a + bi$, what is the value of a ?
- (A) $1/6$ (B) $-1/7$ (C) $7/13$ (D) $-7/85$ (E) $6/85$
31. The symbol $(x)_7$ means that the number is written in base 7; so, for instance, $(23)_7$ equals $2 \times 7 + 3 = 17$ in base 10. Suppose that a certain integer x can be written as a two-digit number in both bases 5 and 6, so that $x = (4z)_5$ and $x = (4y)_6$. What is x (in base 10)?
- (A) 30 (B) 20 (C) 16 (D) 24 (E) 19
32. What is the shape (circle, ellipse, etc.) of the graph of the relation $x^2 + 4x + 6y - y^2 = 8$?
- (A) circle (B) ellipse (C) hyperbola (D) parabola (E) None of these

44. A farmer bought 800 sheep. He sold 400 of them at a markup of 50%. At what markup must he sell the remaining sheep so as to double his investment on the entire transaction?
- (A) 150% (B) 100% (C) 50% (D) 200% (E) 250%
45. Five students took the calculus final exam. The three who passed averaged 80 on the test. The two who failed only averaged 55 points. What was the average for the whole class?
- (A) 72.5 (B) 70 (C) 67.5 (D) 65 (E) 62.5
46. The average on the trigonometry class's final exam was 75. However, one student had a 0 on the exam. If the teacher removes this grade and re-computes the average, she gets 78. How many students took the test?
- (A) 27 (B) 26 (C) 25 (D) 24 (E) 23
47. Suppose that $2^a + 2^b = 3^c + 3^d$, where all of the exponents are integers. How many of a, b, c, d can be negative?
- (A) None (B) 1 (C) 2 (D) 3 (E) 4
48. Suppose that $\ln x$ and $\log_{10} x$ are both integers. How many possible values of x are there?
- (A) 0 (B) 1 (C) 2 (D) 3 (E) ∞
49. If the reciprocal of $(x - 1)$ is $x + \frac{1}{2}$, what is x if $x > 0$?
- (A) -1 (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$ (E) None of these
50. James did the following addition problem: $440 + 340 = 1000$. This problem is done correctly! In what base was James writing his numbers?
- (A) 9 (B) 5 (C) 6 (D) 7 (E) 8

What You Can Do With A Mathematics Major

Occupational opportunities

Actuarial and Insurance	Government	Accountant
Computer & Information Sciences	Investment Analyst	Financial Planner
Researcher	Benefits Specialist	Mathematician
Demographers	Computer Programmer	Cartographer
Data Processor	Navigator	Meteorologist
Applications Programmer	Ecologist	Health
Systems Analyst	Biomedical Engineer	Bio-mathematician
Computer Applications Engineer	Operations Analyst	Operations Research
Control Systems Engineer	Control Systems Engineer	Systems Engineer
Numerical Analyst	Teaching	Business Industry
Statistician	Engineering Analyst	Financial Analyst
Technical Writer	Homeland Security	Communications Engineer

Study in the field of mathematics offers an education with an emphasis on careful problem analysis, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, health and environmental fields, and the academic world require sophisticated mathematical techniques for their solution. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of Alabama offers undergraduate and graduate degrees in Mathematics. Please visit www.ua.edu and refer to the undergraduate and graduate programs for additional information.

Engineering Math Advancement Program

The University of Alabama is offering a new summer program to build math skills for students entering engineering. The Engineering Math Advancement Program (EMAP) is a summer residence class that addresses math and engineering prerequisites for incoming engineering students. The program targets bright students who may not have retained the information learned in high school and provides an opportunity to hone technical abilities before entering college. The goal of E-MAP is to assist entering freshmen in developing a solid background in calculus to succeed in engineering before they start at the University.

Classes are designed around Precalculus Algebra and Trigonometry and incorporate important learning principles to ensure that knowledge is retained and not just memorized. Students develop their skills through hands-on experiences, problem solving teaming exercises, and interaction with engineering professors and instructors through an interdisciplinary Living Laboratory program. Experiments allow students to use simple calculus in engineering applications. The program also involves introducing students to local practicing engineers through work on one or more community service engineering-related activities. E-MAP will reserve 33-40 percent of enrollment space for underrepresented groups. Financial assistance is available based on need. Please visit emap.ua.edu for additional information.