

Alabama State-Wide Mathematics Contest

2002 Ciphering Problems

Round 1

Division 1

Geometry

Question 1

A rectangle whose length is twice its width is inscribed in a circle of radius 10. What is the area of the rectangle?

Answer: 160

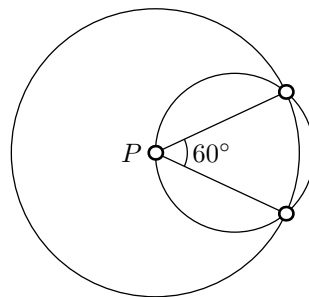
Round 1

Division 1

Geometry

Question 2

P is the center of the larger circle which has radius 9. What is the radius of the smaller circle?



Answer: $3\sqrt{3}$

Round 1

Division 1

Algebra

Question 1

Find the positive value of n so that the difference between the roots of

$$2x^2 + nx - 14$$

is 8.

Answer: 12

Round 1

Division 1

Algebra

Question 2

Find the largest possible value of x in a solution of

$$x^2 + xy = 4$$

$$y^2 + xy = 1$$

Answer: $\frac{4\sqrt{5}}{5}$

Round 1

Division 1

Comprehensive A

Question 1

Find the value of x such that

$$\log_x 100 = \log_8 10000$$

Answer: $2\sqrt{2}$

Round 1

Division 1

Comprehensive A

Question 2

The product of two of the roots of

$$x^3 - kx^2 + 8x - 6$$

is 3. Find the sum of these two roots.

Answer: $\frac{5}{2}$

Round 1

Division 1

Comprehensive B

Question 1

Find the area of the largest circle which can be inscribed in a semicircle of radius 30.

Answer: 225π

Round 1

Division 1

Comprehensive B

Question 2

How many 4-digit telephone extensions can be formed (with 0 allowed in any digit) in which two or more digits are the same?

Answer: 4960

Round 1

Division 1

Team

Question 1

Solve for n :

$$\sqrt{\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdots \frac{n}{n-1}} = 6$$

Answer: 72

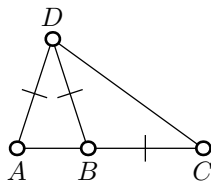
Round 1

Division 1

Team

Question 2

All three triangles are isosceles and $AB = 2$. What is AD ?



Answer: $1 + \sqrt{5}$

Round 2

Division 1

Geometry

Question 1

A right circular cone has a lateral surface area of 65π and a total surface area of 90π . What is its volume?

Answer: 100π

Round 2

Division 1

Geometry

Question 2

A solid regular octahedron has volume $8\sqrt{6}$ cubic units. What is its surface area in square units?

Answer: $24\sqrt{3}$

Round 2

Division 1

Algebra

Question 1

If 99 passengers show up for a flight but the airplane has only 97 seats, in how many different ways can the airline choose which 97 people will fly?

Answer: 4851

Round 2

Division 1

Algebra

Question 2

Solve for x :

$$\frac{x^4 - 1}{x^3 + x^2 + x + 1} = 2 - 3x$$

Answer: $\frac{3}{4}$

Round 2

Division 1

Comprehensive A

Question 1

In a regular polygon, the measure of an interior angle is 18 times the measure of an exterior angle. How many sides does the polygon have?

Answer: 38

Round 2

Division 1

Comprehensive A

Question 2

From 5 students ages 11, 12, 13, 14, and 15, two are randomly chosen. What is the probability that their ages differ by at least 2 years?

Answer: $\frac{3}{5}$

Round 2

Division 1

Comprehensive B

Question 1

Compute

$$\log_2 [\log_4 (\log_{10} 16^4 + \log_{10} 25^8)]$$

Answer: 1

Round 2

Division 1

Comprehensive B

Question 2

One of the roots of

$$x^3 + 5x^2 + qx - 7$$

is the negative of the other. What is q ?

Answer: $-\frac{7}{5}$

Round 2

Division 1

Team

Question 1

In a tennis tournament, the odds that player A will win are 5 to 4 and the odds that player B will win are 1 to 5. What is the probability that A or B will win, expressed as a fraction in lowest terms?

Answer: $\frac{13}{18}$

Round 2

Division 1

Team

Question 2

A function f satisfies

$$f(n^2) = f(n) + 6$$

for $n \geq 2$ and

$$f(2) = 8.$$

Find $f(256)$.

Answer: 26

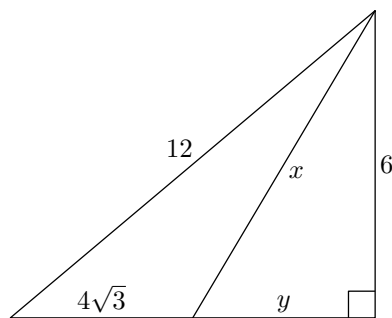
Round 3

Division 1

Geometry

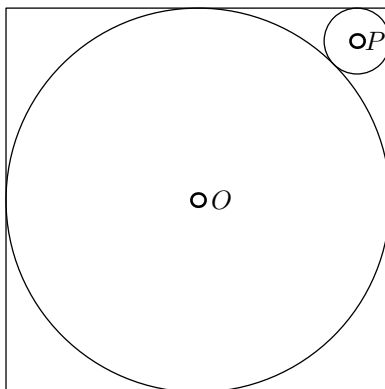
Question 1

Find x .



Answer: $4\sqrt{3}$

The circle O has unit radius and is inscribed in the square. Find the radius of the circle P inscribed as shown.



Answer: $3 - 2\sqrt{2}$

If

$$\frac{1 - x + 6x^2}{x - x^3} = \frac{A}{x} + \frac{B}{1 - x} + \frac{C}{1 + x},$$

then $B = ?$

Answer: 3

Round 3

Division 1

Algebra

Question 2

If $a = \log_{10} 25$ and $b = \log_{10} 36$, what is

$$6^{a/b} + 5^{b/a} ?$$

Answer: 11

Round 3

Division 1

Comprehensive A

Question 1

Solve for x :

$$x^6 + x^3 + 1 = \frac{511}{x^3 - 1}$$

Answer: 2

Round 3

Division 1

Comprehensive A

Question 2

A sphere of radius 2 inscribed in a right circular cone of height 6 contacts the lateral surface of the cone in a circle of what radius?

Answer: $\sqrt{3}$

Round 3

Division 1

Comprehensive B

Question 1

Find the sum of the digits of the number

$$4^{2004}5^{4002}$$

Answer: 10

Round 3

Division 1

Comprehensive B

Question 2

If $i = \sqrt{-1}$, find the real part of the solution of

$$|z| - z = 1 + 2i$$

Answer: $\frac{3}{2}$

Round 3

Division 1

Team

Question 1

If the solution of the inequality

$$2x^2 + px + q < 0$$

is the interval $(\frac{1}{2}, 4)$, then $p + q = ?$

Answer: -5

Round 3

Division 1

Team

Question 2

Find the largest prime factor of

$$2^{18} - 1 .$$

Answer: 73

Round 4

Division 1

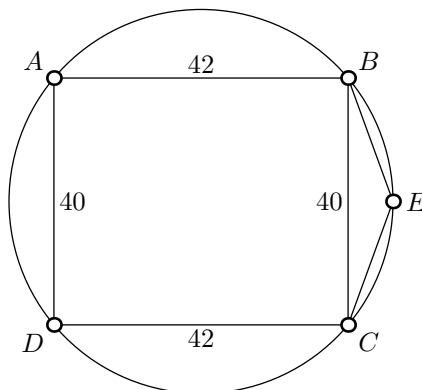
Geometry

Question 1

If the central angle of a sector of a circle is increased by 3 radians, the area of the sector increases by 9. What is the radius of the circle?

Answer: $\sqrt{6}$

Points A , B , C , D , and E are on the circle and $BE = CE$. Find BE .



Answer: $4\sqrt{29}$

The solution of the inequality

$$|\sqrt{x} - \sqrt{2}| < 2\sqrt{2}$$

is an interval of what length?

Answer: 18

Round 4

Division 1

Algebra

Question 2

Jack and Jill visit an animal shelter having 15 cats and 10 dogs. How many ways can Jack take home exactly one pet, either a cat or a dog, and Jill take home exactly two pets, a cat and a dog?

Answer: 3450

Round 4

Division 1

Comprehensive A

Question 1

If $\sin \theta = \frac{3}{5}$ and $\cos \theta < 0$, what is

$\sin 2\theta + \cos 2\theta$?

Answer: $-\frac{17}{25}$

Round 4

Division 1

Comprehensive A

Question 2

Find

$$\lim_{n \rightarrow \infty} 10\sqrt{10}^4\sqrt[4]{10}^8\sqrt[8]{10}^{16}\sqrt[16]{10} \cdots \sqrt[2^n]{10}.$$

Answer: 100

Round 4

Division 1

Comprehensive B

Question 1

In the binomial expansion of

$$(1 + x)^n,$$

the coefficients of x^6 and x^{14} are equal. What is n ?

Answer: 20

Round 4

Division 1

Comprehensive B

Question 2

Three circles of radius 8 are mutually tangent to each other with points of tangency A , B , and C . What is the area of $\triangle ABC$?

Answer: $16\sqrt{3}$

Round 4

Division 1

Team

Question 1

Solve for x :

$$(5^x + 5^x + 5^x + 5^x + 5^x)^{90} = 125^{50}$$

Answer: $\frac{2}{3}$

Round 4

Division 1

Team

Question 2

Given seven different points on a circle, how many different convex polygons can be formed using subsets of these points as vertices?

Answer: 99

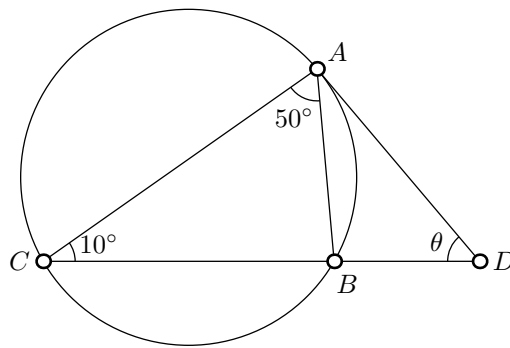
Round 5

Division 1

Geometry

Question 1

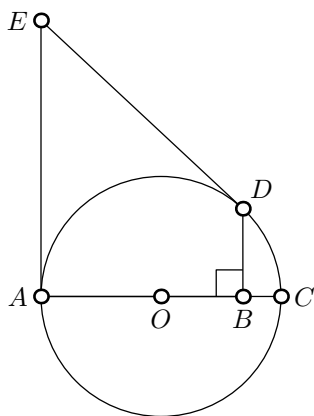
If \overline{DA} is tangent to the circle at A , $m\angle ACB = 10^\circ$, and $m\angle CAB = 50^\circ$, find the degree measure of $\angle\theta$.



Answer: 110

If \overline{EA} and \overline{ED} are tangent to the circle at A and D , \overline{DB} is perpendicular to diameter \overline{AC}

of circle O , and $\frac{EA}{AO} = 12$, find $\frac{AB}{BC}$.



Answer: 144

Find the sum

$$3 + 8 + 13 + 18 + 23 + \cdots + 328 .$$

Answer: 10,923

Round 5

Division 1

Algebra

Question 2

In 8 years, a girl will be $\frac{5}{8}$ as old as her father was 8 years ago, whereas in 4 years she will be $\frac{4}{9}$ as old as he was 4 years ago. How much older in years is the father than the girl?

Answer: 28

Round 5

Division 1

Comprehensive A

Question 1

If

$$x + y = 12$$

and

$$x^2 + 3xy + 2y^2 = 180,$$

find

$$4x + 2y.$$

Answer: 42

Round 5

Division 1

Comprehensive A

Question 2

If $i = \sqrt{-1}$, then

$$(1 + i)^{17} = a + bi$$

where $a + b = ?$

Answer: 512

Round 5

Division 1

Comprehensive B

Question 1

In how many ways can a police lineup be made of 6 suspects if 2 of the suspects cannot stand together?

Answer: 480

Round 5

Division 1

Comprehensive B

Question 2

If

$$(x^2 + y^2)^3 = (x^3 + y^3)^2$$

and

$$xy \neq 0,$$

then

$$\left(\frac{x}{y} + \frac{y}{x}\right)^2 = ?$$

Answer: $\frac{4}{9}$

Round 5

Division 1

Team

Question 1

A teacher has a class of 9 students: 3 girls and 6 boys. How many ways can she seat her students in a line of 9 chairs if between any 2 girls there are exactly 2 boys?

Answer: 12,960

Round 5

Division 1

Team

Question 2

If A , B , and C are positive integers so that

$$\frac{31}{4} = A + \frac{1}{B + \frac{1}{C+1}}$$

What is $A^2 + B^2 + C^2$?

Answer: 54

Round 1

Division 2

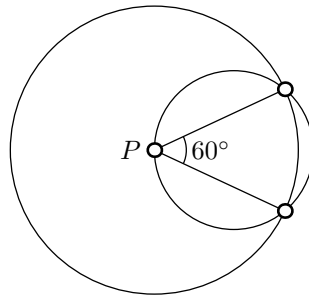
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A rectangle whose length is twice its width is inscribed in a circle of radius 10. What is the area of the rectangle?

Answer: 160

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Division 2

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Round 1

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Round 1

Division 2

Comprehensive B

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Round 1

Division 2

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Round 1

Division 2

Team

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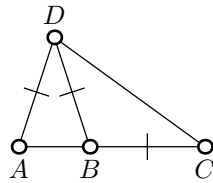
Round 1

Division 2

Team

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Round 2

Division 2

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Round 2

Division 2

Geometry

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Round 2

Division 2

Algebra

Question 1

If 99 passengers show up for a flight but the airplane has only 97 seats, in how many different ways can the airline choose which 97 people will fly?

Answer: 4851

Round 2

Division 2

Algebra

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Answer: $\frac{3}{4}$

Round 2

Division 2

Comprehensive A

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Round 2

Division 2

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Round 2

Division 2

Comprehensive B

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Round 2

Division 2

Comprehensive B

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Round 2

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Round 2

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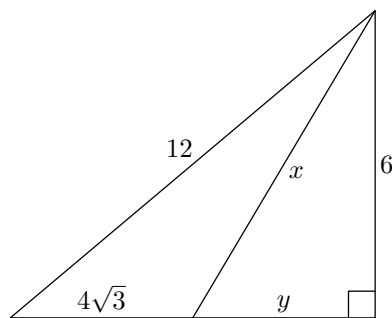
Round 3

Division 2

Geometry

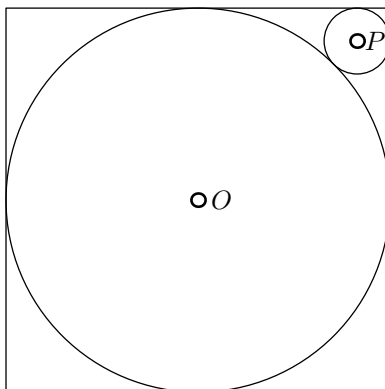
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Algebra

Question 2

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Round 3

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Answer: 2

Round 3

Division 2

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Round 3

Division 2

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Round 3

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Comprehensive B

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Round 3

Division 2

Team

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Round 3

Division 2

Team

Question 2

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Round 4

Division 2

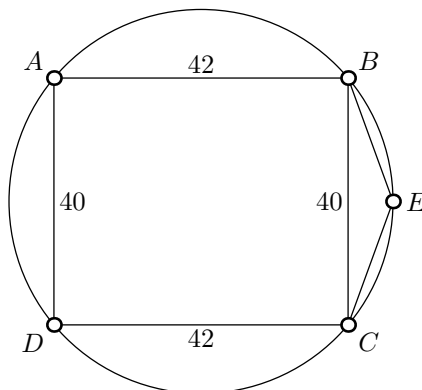
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Algebra

Question 2

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Round 4

Division 2

Comprehensive A

Question 1

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Round 4

Division 2

Comprehensive A

Question 2

Find

$$\lim_{n \rightarrow \infty} 10\sqrt{10}^4\sqrt[4]{10}^8\sqrt[8]{10}^{16}\sqrt[16]{10} \cdots \sqrt[2^n]{10}.$$

Answer: 100

Round 4

Division 2

Comprehensive B

Question 1

In the binomial expansion of

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the coefficients of x^6 and x^{14} are equal. What is n ?

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Round 4

Division 2

Comprehensive B

Question 2

Three circles of radius 8 are mutually tangent to each other with points of tangency A , B , and C . What is the area of $\triangle ABC$?

Answer: $16\sqrt{3}$

Round 4

Division 2

Team

Question 1

Solve for x :

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Round 4

Division 2

Team

Question 2

Given seven different points on a circle, how many different convex polygons can be formed using subsets of these points as vertices?

Answer: 99

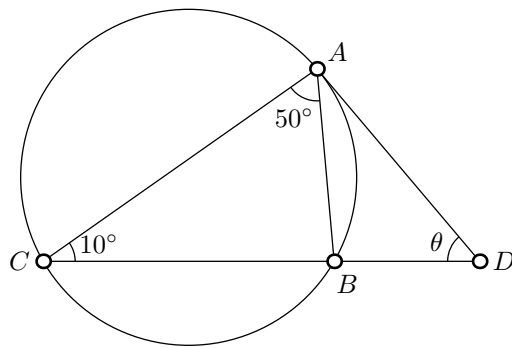
Round 5

Division 2

Geometry

Question 1

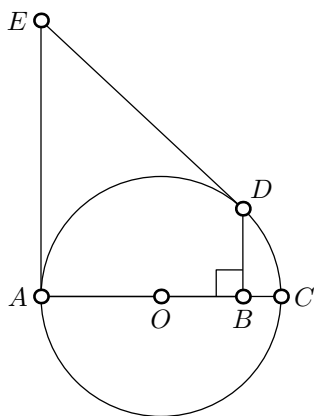
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of circle O , and $\frac{EA}{AO} = 12$, find $\frac{AB}{BC}$.



Answer: 144

Find the sum

$$3 + 8 + 13 + 18 + 23 + \cdots + 328 .$$

Answer: 10,923

Round 5

Division 2

Algebra

Question 2

In 8 years, a girl will be $\frac{5}{8}$ as old as her father was 8 years ago, whereas in 4 years she will be $\frac{4}{9}$ as old as he was 4 years ago. How much older in years is the father than the girl?

Answer: 28

Round 5

Division 2

Comprehensive A

Question 1

If

$$x + y = 12$$

and

$$x^2 + 3xy + 2y^2 = 180,$$

find

$$4x + 2y.$$

Answer: 42

Round 5

Division 2

Comprehensive A

Question 2

If $i = \sqrt{-1}$, then

$$(1 + i)^{17} = a + bi$$

where $a + b = ?$

Answer: 512

Round 5

Division 2

Comprehensive B

Question 1

In how many ways can a police lineup be made of 6 suspects if 2 of the suspects cannot stand together?

Answer: 480

Round 5

Division 2

Comprehensive B

Question 2

If

$$(x^2 + y^2)^3 = (x^3 + y^3)^2$$

and

$$xy \neq 0,$$

then

$$\left(\frac{x}{y} + \frac{y}{x}\right)^2 = ?$$

Answer: $\frac{4}{9}$

Round 5

Division 2

Team

Question 1

A teacher has a class of 9 students: 3 girls and 6 boys. How many ways can she seat her students in a line of 9 chairs if between any 2 girls there are exactly 2 boys?

Answer: 12,960

Round 5

Division 2

Team

Question 2

If A , B , and C are positive integers so that

$$\frac{31}{4} = A + \frac{1}{B + \frac{1}{C+1}}$$

What is $A^2 + B^2 + C^2$?

Answer: 54

Round 1

Division 3

Geometry

Question 1

If the area of a circle is to be decreased by 25%, its radius should be multiplied by what factor?

Answer: $\frac{\sqrt{3}}{2}$

Round 1

Division 3

Geometry

Question 2

A rectangle whose length is twice its width is inscribed in a circle of radius 10. What is the area of the rectangle?

Answer: 160

Round 1

Division 3

Algebra

Question 1

$$4^{-2} - 2^{-3} = ?$$

Answer: $-\frac{1}{16}$

Round 1

Division 3

Algebra

Question 2

Find the positive value of n so that the difference between the roots of

$$2x^2 + nx - 14$$

is 8.

Answer: 12

Round 1

Division 3

Comprehensive A

Question 1

The sides of a pentagon measure 2, 3, 4, 5, and 6. Find the perimeter of a similar pentagon whose longest side measures 15.

Answer: 50

Round 1

Division 3

Comprehensive A

Question 2

Find the value of x such that

$$\log_x 100 = \log_8 10000$$

Answer: $2\sqrt{2}$

Round 1

Division 3

Comprehensive B

Question 1

Find the sum of the solutions of:

$$10x^2 + 11x - 6 = 0$$

Answer: $-\frac{11}{10}$

Round 1

Division 3

Comprehensive B

Question 2

Find the area of the largest circle which can be inscribed in a semicircle of radius 30.

Answer: 225π

Round 1

Division 3

Team

Question 1

$$\log_{1/8} \sqrt[4]{4096} = ?$$

Answer: -1

Round 1

Division 3

Team

Question 2

Solve for n :

$$\sqrt{\frac{3}{2} \cdot \frac{4}{3} \cdot \frac{5}{4} \cdots \frac{n}{n-1}} = 6$$

Answer: 72

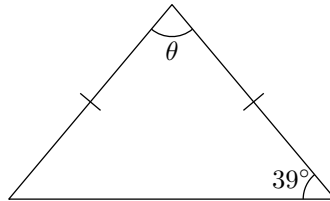
Round 2

Division 3

Geometry

Question 1

What is the degree measure of angle θ in the isosceles triangle?



Answer: 102

Round 2

Division 3

Geometry

Question 2

A right circular cone has a lateral surface area of 65π and a total surface area of 90π . What is its volume?

Answer: 100π

Round 2

Division 3

Algebra

Question 1

$$\frac{1}{2 + \frac{3}{4 + \frac{5}{6}}} = ?$$

Answer: $\frac{29}{76}$

Round 2

Division 3

Algebra

Question 2

If 99 passengers show up for a flight but the airplane has only 97 seats, in how many different ways can the airline choose which 97 people will fly?

Answer: 4851

Round 2

Division 3

Comprehensive A

Question 1

If $x \neq 0$, $y \neq 0$, and

$$3xy^2 = 4xy$$

then $y = ?$

Answer: $\frac{4}{3}$

Round 2

Division 3

Comprehensive A

Question 2

In a regular polygon, the measure of an interior angle is 18 times the measure of an exterior angle. How many sides does the polygon have?

Answer: 38

Round 2

Division 3

Comprehensive B

Question 1

Find the perimeter of the triangle in the xy -plane formed by the x -axis, the y -axis, and the line whose equation is $4y + 3x = 12$.

Answer: 12

Round 2

Division 3

Comprehensive B

Question 2

Compute

$$\log_2 [\log_4 (\log_{10} 16^4 + \log_{10} 25^8)]$$

Answer: 1

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Division 3

Team

Question 1

If

$$x = \frac{\sqrt{17} + \sqrt{13}}{2}$$

and

$$y = \frac{\sqrt{17} - \sqrt{13}}{2}$$

then what is

$$x^2 - xy + y^2 ?$$

Answer: 14

Round 2

Division 3

Team

Question 2

In a tennis tournament, the odds that player A will win are 5 to 4 and the odds that player B will win are 1 to 5. What is the probability that A or B will win, expressed as a fraction in lowest terms?

Answer: $\frac{13}{18}$

Round 3

Division 3

Geometry

Question 1

What is the measure of an altitude of an equilateral triangle whose sides each measure 10?

Answer: $5\sqrt{3}$

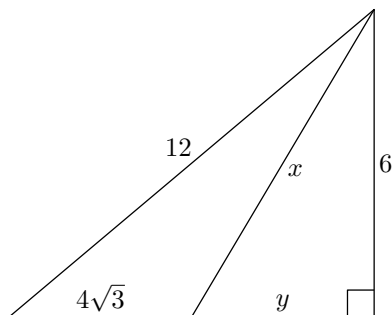
Round 3

Division 3

Geometry

Question 2

Find x .



Answer: $4\sqrt{3}$

Round 3

Division 3

Algebra

Question 1

Solve for x :

$$\sqrt[3]{\sqrt{x}} = 2$$

Answer: 64

Round 3

Division 3

Algebra

Question 2

If

$$\frac{1 - x + 6x^2}{x - x^3} = \frac{A}{x} + \frac{B}{1 - x} + \frac{C}{1 + x},$$

then $B = ?$

Answer: 3

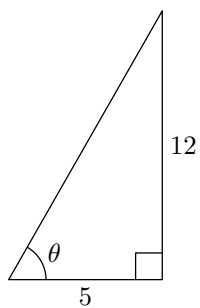
Round 3

Division 3

Comprehensive A

Question 1

$$\csc \theta = ?$$



Answer: $\frac{13}{12}$

Round 3

Division 3

Comprehensive A

Question 2

Solve for x :

$$x^6 + x^3 + 1 = \frac{511}{x^3 - 1}$$

Answer: 2

Round 3

Division 3

Comprehensive B

Question 1

A square inscribed in a circle has area 2. What is the area of the circle?

Answer: π

Round 3

Division 3

Comprehensive B

Question 2

Find the sum of the digits of the number

$$4^{2004}5^{4002}$$

Answer: 10

Round 3

Division 3

Team

Question 1

An equilateral triangle whose sides each measure $2\sqrt{3}$ is inscribed in a circle. Find the area of the circle.

Answer: 4π

Round 3

Division 3

Team

Question 2

If the solution of the inequality

$$2x^2 + px + q < 0$$

is the interval $(\frac{1}{2}, 4)$, then $p + q = ?$

Answer: -5

Round 4

Division 3

Geometry

Question 1

If the perimeter of a square is 10 units, the area of the square is how many square units?

Answer: $\frac{25}{4}$

Round 4

Division 3

Geometry

Question 2

If the central angle of a sector of a circle is increased by 3 radians, the area of the sector increases by 9. What is the radius of the circle?

Answer: $\sqrt{6}$

Round 4

Division 3

Algebra

Question 1

The system

$$3x + 4y = 5$$

$$kx - 8y = -9$$

has no solution if $k = ?$

Answer: -6

Round 4

Division 3

Algebra

Question 2

The solution of the inequality

$$|\sqrt{x} - \sqrt{2}| < 2\sqrt{2}$$

is an interval of what length?

Answer: 18

Round 4

Division 3

Comprehensive A

Question 1

Solve for x :

$$\frac{x+2}{x-3} = \frac{2}{3}$$

Answer: -12

Round 4

Division 3

Comprehensive A

Question 2

If $\sin \theta = \frac{3}{5}$ and $\cos \theta < 0$, what is

$$\sin 2\theta + \cos 2\theta ?$$

Answer: $-\frac{17}{25}$

Round 4

Division 3

Comprehensive B

Question 1

$$\left(\frac{1}{8}\right)^{-4/3} = ?$$

Answer: 16

Round 4

Division 3

Comprehensive B

Question 2

In the binomial expansion of

$$(1 + x)^n,$$

the coefficients of x^6 and x^{14} are equal. What is n ?

Answer: 20

Round 4

Division 3

Team

Question 1

What is the sum of the degree measures of the interior angles of a convex pentagon?

Answer: 540

Round 4

Division 3

Team

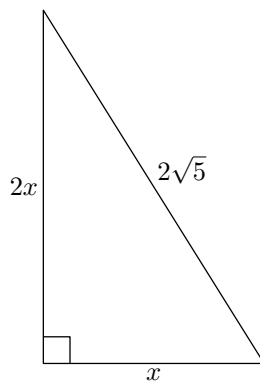
Question 2

Solve for x :

$$(5^x + 5^x + 5^x + 5^x + 5^x)^{90} = 125^{50}$$

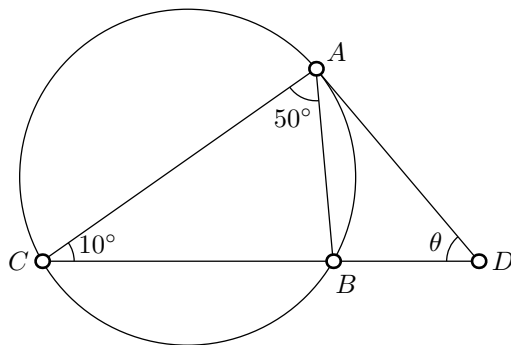
Answer: $\frac{2}{3}$

Find x in the right triangle:



Answer: 2

If \overline{DA} is tangent to the circle at A , $m\angle ACB = 10^\circ$, and $m\angle CAB = 50^\circ$, find the degree measure of $\angle\theta$.



Answer: 110

Round 5

Division 3

Algebra

Question 1

Find a negative number which is 12 less than its square.

Answer: -3

Round 5

Division 3

Algebra

Question 2

Find the sum

$$3 + 8 + 13 + 18 + 23 + \cdots + 328 .$$

Answer: 10,923

Round 5

Division 3

Comprehensive A

Question 1

A circle of what radius has an area equal to the combined area of two circles with radii 2 and 4?

Answer: $2\sqrt{5}$

Round 5

Division 3

Comprehensive A

Question 2

If

$$x + y = 12$$

and

$$x^2 + 3xy + 2y^2 = 180,$$

find

$$4x + 2y.$$

Answer: 42

Round 5

Division 3

Comprehensive B

Question 1

Solve for n :

$$n! = 90(n - 2)!$$

Answer: 10

Round 5

Division 3

Comprehensive B

Question 2

In how many ways can a police lineup be made of 6 suspects if 2 of the suspects cannot stand together?

Answer: 480

Round 5

Division 3

Team

Question 1

Find the largest value of x so that

$$\sqrt[4]{x} = \frac{18}{11 - \sqrt[4]{x}}$$

Answer: 6561

Round 5

Division 3

Team

Question 2

A teacher has a class of 9 students: 3 girls and 6 boys. How many ways can she seat her students in a line of 9 chairs if between any 2 girls there are exactly 2 boys?

Answer: 12,960