

Alabama State-Wide Mathematics Contest

2003 Ciphering Problems

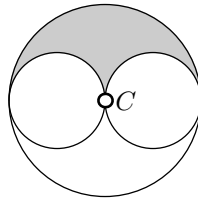
Round 1

Division 1

Geometry

Question 1

The area of the shaded region is 2π . What is the area of the large circle having center C ?



Answer: 8π

Round 1

Division 1

Geometry

Question 2

A $30^\circ - 120^\circ - 30^\circ$ isosceles triangle with base measuring 1 is inscribed in a circle. What is the area of the circle?

Answer: $\frac{\pi}{3}$

Round 1

Division 1

Algebra

Question 1

If the line through points $(m, -16)$ and $(9, m)$ has slope m , find m .

Answer: 4

Round 1

Division 1

Algebra

Question 2

If

$$k^{\log_2 5} = 32$$

find

$$k^{(\log_2 5)^2}$$

Answer: 3125

Round 1

Division 1

Comprehensive A

Question 1

In the x - y plane, 3 vertices of a square are the points $(4, 0)$, $(0, 3)$, and $(-3, -1)$. The 4th vertex is (a, b) where $a + b = ?$

Answer: -3

Round 1

Division 1

Comprehensive A

Question 2

Find the least positive integer n such that

$$\sqrt{n+1} - \sqrt{n} < \frac{1}{10}$$

Answer: 25

Round 1

Division 1

Comprehensive B

Question 1

Solve for x :

$$3 \log_2 x + \log_2 27 = 3$$

Answer: $\frac{2}{3}$

Round 1

Division 1

Comprehensive B

Question 2

If a and b are the roots of $x^2 - 60x + 598$, find $a^2 + b^2$.

Answer: 2404

Round 1

Division 1

Team

Question 1

The internal angles of a (convex) pentagon are in the ratio 1:2:3:4:5. What is the degree measure of the smallest internal angle in the pentagon?

Answer: 36

Round 1

Division 1

Team

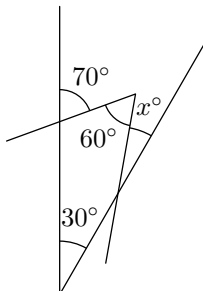
Question 2

Find the final digit of

$$(1! + 3! + 5! + \dots + 99!)(2! + 4! + 6! + \dots + 98!)$$

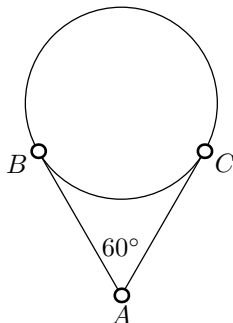
Answer: 2

What is x ?



Answer: 20

If \overline{AB} and \overline{AC} are tangent to the circle of radius 1 at B and C respectively, what is the area of $\triangle ABC$?



Answer: $\frac{3\sqrt{3}}{4}$

Round 2

Division 1

Algebra

Question 1

How many ways are there to distribute 4 different teddy bears to 3 children if each child gets at least 1 teddy bear?

Answer: 36

Round 2

Division 1

Algebra

Question 2

Solve for x :

$$\left(\sqrt{4\sqrt{x} - \sqrt{12x + \sqrt{8x - 1}}} \right) \cdot$$

$$\left(\sqrt{4\sqrt{x} + \sqrt{12x + \sqrt{8x - 1}}} \right) \cdot$$

$$\left(\sqrt{4x + \sqrt{8x - 1}} \right) = 15$$

Answer: 4

Round 2

Division 1

Comprehensive A

Question 1

The intersection of a sphere of radius 10 with a plane having distance 7 from the center of the sphere is a circle of what area?

Answer: 51π

Round 2

Division 1

Comprehensive A

Question 2

If $x^3 + ax^2 + x + b$ is divisible by $x^2 - 3x + 2$, what is a ?

Answer: $-\frac{8}{3}$

Round 2

Division 1

Comprehensive B

Question 1

What is the greatest value of c such that the system of equations

$$x^2 + y^2 = 25$$

$$x + y = c$$

has a (real) solution?

Answer: $5\sqrt{2}$

Round 2

Division 1

Comprehensive B

Question 2

If

$$\tan \frac{\theta}{2} = \frac{2}{3}$$

what is $\sin \theta$?

Answer: $\frac{12}{13}$

Round 2

Division 1

Team

Question 1

A parabola's focus F is the midpoint of a side of a square of area 1. Its directrix is the line containing the side of the square opposite the side containing the focus. The parabola intersects the square at points P and Q where $|PF| = |QF| = ?$

Answer: $\frac{5}{8}$

Round 2

Division 1

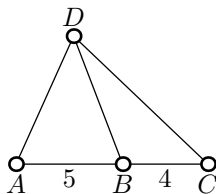
Team

Question 2

In rectangle $ABCD$: $AB = 20$, $AD = 15$. A point P is inside the rectangle such that $AP = 12$ and $BP = 16$. Find the distance between P and \overline{DC} .

Answer: $\frac{27}{5}$

If $\triangle BCD$ has area 12, what is the area of $\triangle ABD$?



Answer: 15

A right circular cylindrical tank 3 feet long and 4 feet in diameter is placed with its axis horizontal. The tank is filled to a depth of 1 foot. How many cubic feet of liquid does it contain?

Answer: $4\pi - 3\sqrt{3}$

Round 3

Division 1

Algebra

Question 1

If $x \neq 0$ and $y \neq 0$ but

$$6x^2y^3 + 27xy^4 = 0$$

find

$$\frac{x}{y}$$

Answer: $-\frac{9}{2}$

Round 3

Division 1

Algebra

Question 2

If

$$x^6 + x^4 + x^2 - 1 = 0$$

find

$$x^8 + 2x^6 + 2x^4$$

Answer: 1

Round 3

Division 1

Comprehensive A

Question 1

Find the sum of the solutions of

$$\sin 2x + \cos x = 0$$

for

$$0 \leq x < 2\pi$$

Answer: 5π

Round 3

Division 1

Comprehensive A

Question 2

How many (positive, whole number) factors does 1,000,000 have (including 1)?

Answer: 49

Round 3

Division 1

Comprehensive B

Question 1

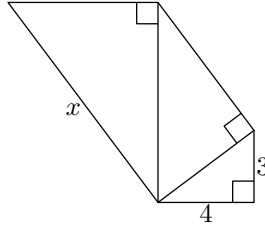
How many integer solutions does the equation

$$1 \leq |x + 2| \leq 4$$

have?

Answer: 8

The three right triangles are similar. What is the largest possible value of x ?



Answer: $\frac{125}{9}$

Two numbers are such that their difference, sum, and product are to each other as $1 : 7 : 36$ respectively. Find the sum of the two numbers.

Answer: 21

In the sequence

$$\{a_n\}_{n=1}^{\infty} = \{1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \dots\}$$

what is a_{130} ?

Answer: 16

Round 4

Division 1

Geometry

Question 1

Two solid iron spheres of radii 1 meter and 2 meters are melted down and the resulting iron used to cast a solid right circular cylinder with a height of 1 meter. What is the radius of the base of the cylinder in meters?

Answer: $2\sqrt{3}$

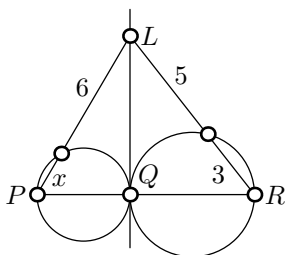
Round 4

Division 1

Geometry

Question 2

The point L is on the common internal tangent of the circles, of which \overline{PQ} and \overline{QR} are diameters. What is x ?



Answer: $\frac{2}{3}$

Round 4

Division 1

Algebra

Question 1

Find x such that

$$\sum_{n=1}^{\infty} x^n = 5$$

Answer: $\frac{5}{6}$

Round 4

Division 1

Algebra

Question 2

If i denotes $\sqrt{-1}$ and for some real y the real part of $(1 + iy)^3$ is -26 , what is $|y|$?

Answer: 3

Round 4

Division 1

Comprehensive A

Question 1

How many ways are there to choose 1 dessert and 3 side orders from a menu having 3 desserts and 10 side orders?

Answer: 360

Round 4

Division 1

Comprehensive A

Question 2

The vertices of an inscribed triangle divide a circle into three arcs whose measures are in the ratio $3 : 4 : 5$. What is the degree measure of the largest angle of the triangle?

Answer: 75

Round 4

Division 1

Comprehensive B

Question 1

Two lines each tangent to a circle of radius 2 intersect at right angles at a point P . What is the distance between P and the center of the circle?

Answer: $2\sqrt{2}$

Round 4

Division 1

Comprehensive B

Question 2

Five (six-sided) dice are tossed. What is the probability that the five numbers shown will be different (as a fraction in lowest terms)?

Answer: $\frac{5}{54}$

Round 4

Division 1

Team

Question 1

Find the minimum value of $x^4 - 6x^2 + 2$ for $-\infty < x < \infty$.

Answer: -7

Round 4

Division 1

Team

Question 2

How many ordered triples of positive integers (r, s, t) satisfy $r + s + t = 52$?

Answer: 1275

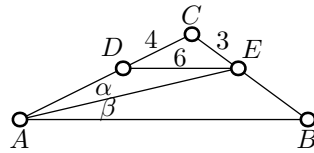
Round 5

Division 1

Geometry

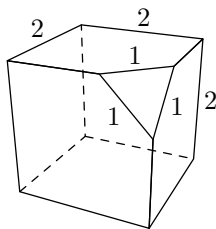
Question 1

If $m \angle \alpha = m \angle \beta$ and $\overline{DE} \parallel \overline{AB}$, what is EB ?



Answer: $\frac{9}{2}$

A corner is cut off of a solid cube as shown. What is the volume of the piece removed from the cube?



Answer: $\frac{\sqrt{2}}{24}$

The quotient and difference of two numbers are both equal to 9. What is the larger of the two numbers?

Answer: $\frac{81}{8}$

Round 5

Division 1

Algebra

Question 2

Find the solution of

$$x + 2\sqrt{|x|} + 1 = 0$$

Answer: $-3 - 2\sqrt{2}$

Round 5

Division 1

Comprehensive A

Question 1

A diagonal of a face of a cube has length 4. What is the length of a diagonal of the cube?

Answer: $2\sqrt{6}$

Round 5

Division 1

Comprehensive A

Question 2

If $\sin \theta + \cos \theta = \frac{1}{4}$, find $\sin 2\theta$.

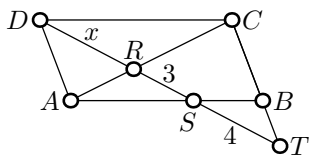
Answer: $-\frac{15}{16}$

Find the sum of the digits of

$$10^{2003} - 10^{1003}$$

Answer: 9000

If $\square ABCD$ is a parallelogram; R is on diagonal AC ; points C , B , and T are collinear; and points D , R , S , and T are collinear, find x .



Answer: $\sqrt{21}$

Round 5

Division 1

Team

Question 1

If

$$xy = 6$$

$$xz = 8$$

$$yz = 12$$

find

$$x^2 + y^2 + z^2$$

Answer: 29

Round 5

Division 1

Team

Question 2

Find the smallest positive integer n such that

$$(n + 1) + (n + 2) + \dots + (n + 23)$$

is a perfect square.

Answer: 11

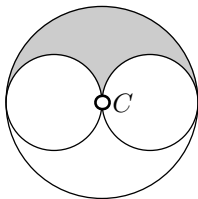
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Division 2

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Division 2

Algebra

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Round 1

Division 2

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Answer: 3125

Round 1

Division 2

Comprehensive A

Question 1

Solve for x :

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Answer: $\frac{2}{3}$

Round 1

Division 2

Comprehensive A

Question 2

In the x - y plane, 3 vertices of a square are the points $(4, 0)$, $(0, 3)$, and $(-3, -1)$. The 4th vertex is (a, b) where $a + b = ?$

Answer: -3

Round 1

Division 2

Comprehensive B

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Round 1

Division 2

Comprehensive B

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Round 1

Division 2

Team

Question 1

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Answer: 36

Round 1

Division 2

Team

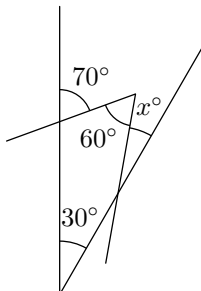
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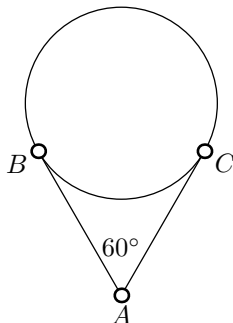
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Answer: 20

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Round 2

Division 2

Algebra

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Round 2

Division 2

Algebra

Question 2

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Round 2

Division 2

Comprehensive A

Question 1

The intersection of a sphere of radius 10 with a plane having distance 7 from the center of the sphere is a circle of what area?

Answer: 51π

Round 2

Division 2

Comprehensive A

Question 2

If $x^3 + ax^2 + x + b$ is divisible by $x^2 - 3x + 2$, what is a ?

Answer: $-\frac{8}{3}$

Round 2

Division 2

Comprehensive B

Question 1

If the polynomial

$$f(x) = x^2 - 13x + 10$$

is divided by the polynomial

$$g(x) = x - 2$$

what is the remainder?

Answer: -12

Round 2

Division 2

Comprehensive B

Question 2

What is the greatest value of c such that the system of equations

$$x^2 + y^2 = 25$$

$$x + y = c$$

has a (real) solution?

Answer: $5\sqrt{2}$

Round 2

Division 2

Team

Question 1

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Round 2

Division 2

Team

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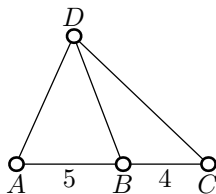
Round 3

Division 2

Geometry

Question 1

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Answer: 15

Round 3

Division 2

Geometry

Question 2

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Answer: $4\pi - 3\sqrt{3}$

Round 3

Division 2

Algebra

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If $x \neq 0$ and $y \neq 0$ but

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Round 3

Division 2

Algebra

Question 2

If

$$x^6 + x^4 + x^2 - 1 = 0$$

find

$$x^8 + 2x^6 + 2x^4$$

Answer: 1

Round 3

Division 2

Comprehensive A

Question 1

Find the sum of the solutions of

$$\sin 2x + \cos x = 0$$

for

$$0 \leq x < 2\pi$$

Answer: 5π

Round 3

Division 2

Comprehensive A

Question 2

How many (positive, whole number) factors does 1,000,000 have (including 1)?

Answer: 49

Round 3

Division 2

Comprehensive B

Question 1

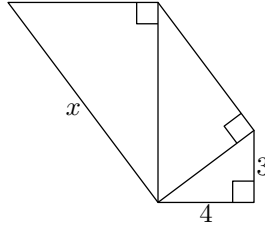
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Answer: 8

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Answer: $\frac{125}{9}$

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Answer: 21

In the sequence

$$\{a_n\}_{n=1}^{\infty} = \{1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \dots\}$$

what is a_{130} ?

Answer: 16

Round 4

Division 2

Geometry

Question 1

The angles of a triangle measure x° , $5x^\circ$, and $6x^\circ$. What is x ?

Answer: 15

Round 4

Division 2

Geometry

Question 2

Two solid iron spheres of radii 1 meter and 2 meters are melted down and the resulting iron used to cast a solid right circular cylinder with a height of 1 meter. What is the radius of the base of the cylinder in meters?

Answer: $2\sqrt{3}$

Round 4

Division 2

Algebra

Question 1

Find x such that

$$\sum_{n=1}^{\infty} x^n = 5$$

Answer: $\frac{5}{6}$

Round 4

Division 2

Algebra

Question 2

If i denotes $\sqrt{-1}$ and for some real y the real part of $(1 + iy)^3$ is -26 , what is $|y|$?

Answer: 3

Round 4

Division 2

Comprehensive A

Question 1

How many ways are there to choose 1 dessert and 3 side orders from a menu having 3 desserts and 10 side orders?

Answer: 360

Round 4

Division 2

Comprehensive A

Question 2

The vertices of an inscribed triangle divide a circle into three arcs whose measures are in the ratio $3 : 4 : 5$. What is the degree measure of the largest angle of the triangle?

Answer: 75

Round 4

Division 2

Comprehensive B

Question 1

Two lines each tangent to a circle of radius 2 intersect at right angles at a point P . What is the distance between P and the center of the circle?

Answer: $2\sqrt{2}$

Round 4

Division 2

Comprehensive B

Question 2

Five (six-sided) dice are tossed. What is the probability that the five numbers shown will be different (as a fraction in lowest terms)?

Answer: $\frac{5}{54}$

Round 4

Division 2

Team

Question 1

Find the minimum value of $x^4 - 6x^2 + 2$ for $-\infty < x < \infty$.

Answer: -7

Round 4

Division 2

Team

Question 2

How many ordered triples of positive integers (r, s, t) satisfy $r + s + t = 52$?

Answer: 1275

Round 5

Division 2

Geometry

Question 1

The surface areas of two cubes are in a ratio of 4 : 9. If V_1 is the volume of the smaller cube and V_2 the volume of the larger cube, what is $\frac{V_1}{V_2}$?

Answer: $\frac{8}{27}$

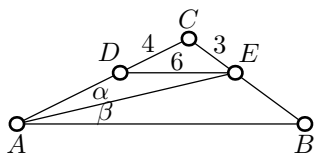
Round 5

Division 2

Geometry

Question 2

If $m \angle \alpha = m \angle \beta$ and $\overline{DE} \parallel \overline{AB}$, what is EB ?



Answer: $\frac{9}{2}$

Round 5

Division 2

Algebra

Question 1

Find a negative solution of

$$|22x + 1| = 19$$

Answer: $-\frac{10}{11}$

Round 5

Division 2

Algebra

Question 2

The quotient and difference of two numbers are both equal to 9. What is the larger of the two numbers?

Answer: $\frac{81}{8}$

Round 5

Division 2

Comprehensive A

Question 1

A diagonal of a face of a cube has length 4. What is the length of a diagonal of the cube?

Answer: $2\sqrt{6}$

Round 5

Division 2

Comprehensive A

Question 2

If $\sin \theta + \cos \theta = \frac{1}{4}$, find $\sin 2\theta$.

Answer: $-\frac{15}{16}$

Round 5

Division 2

Comprehensive B

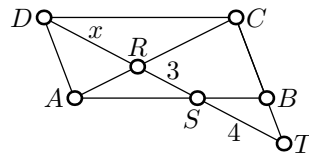
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Find the sum of the digits of

$$10^{2003} - 10^{1003}$$

Answer: 9000

If $\square ABCD$ is a parallelogram; R is on diagonal AC ; points C , B , and T are collinear; and points D , R , S , and T are collinear, find x .



Answer: $\sqrt{21}$

If

$$xy = 6$$

$$xz = 8$$

$$yz = 12$$

find

$$x^2 + y^2 + z^2$$

Answer: 29

Round 5

Division 2

Team

Question 2

Find the smallest positive integer n such that

$$(n + 1) + (n + 2) + \dots + (n + 23)$$

is a perfect square.

Answer: 11

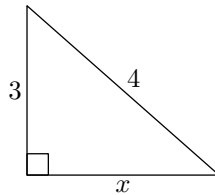
Round 1

Division 3

Geometry

Question 1

What is x ?



Answer: $\sqrt{7}$

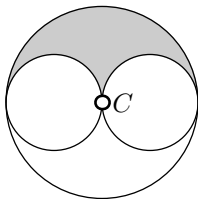
Round 1

Division 3

Geometry

Question 2

The area of the shaded region is 2π . What is the area of the large circle having center C ?



Answer: 8π

Round 1

Division 3

Algebra

Question 1

$$\sqrt{16^5} = ?$$

Answer: 1024

Round 1

Division 3

Algebra

Question 2

If the line through points $(m, -16)$ and $(9, m)$ has slope m , find m .

Answer: 4

Round 1

Division 3

Comprehensive A

Question 1

Solve for x :

$$\sqrt{2+x} = 2\sqrt{x}$$

Answer: $\frac{2}{3}$

Round 1

Division 3

Comprehensive A

Question 2

In the x - y plane, 3 vertices of a square are the points $(4, 0)$, $(0, 3)$, and $(-3, -1)$. The 4th vertex is (a, b) where $a + b = ?$

Answer: -3

Round 1

Division 3

Comprehensive B

Question 1

An interior angle of a parallelogram measures 15° . What is the degree measure of the largest interior angle of the parallelogram?

Answer: 165

Round 1

Division 3

Comprehensive B

Question 2

Solve for x :

$$3\log_2 x + \log_2 27 = 3$$

Answer: $\frac{2}{3}$

Round 1

Division 3

Team

Question 1

Solve for x :

$$\sqrt{x-3} - \sqrt{x+3} = -2$$

Answer: $\frac{13}{4}$

Round 1

Division 3

Team

Question 2

The internal angles of a (convex) pentagon are in the ratio 1:2:3:4:5. What is the degree measure of the smallest internal angle in the pentagon?

Answer: 36

Round 2

Division 3

Geometry

Question 1

A sector of a circle of radius 3 has area 2π . What is the degree measure of its central angle?

Answer: 80

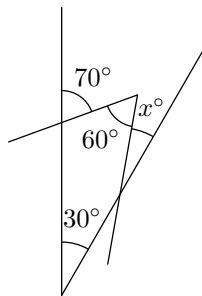
Round 2

Division 3

Geometry

Question 2

What is x ?



Answer: 20

Round 2

Division 3

Algebra

Question 1

Solve for x :

$$\frac{1}{2 + \frac{1}{x}} = \frac{x}{6}$$

Answer: $\frac{5}{2}$

Round 2

Division 3

Algebra

Question 2

How many ways are there to distribute 4 different teddy bears to 3 children if each child gets at least 1 teddy bear?

Answer: 36

Round 2

Division 3

Comprehensive A

Question 1

The line perpendicular to the line whose equation is

$$x + 2y = 9$$

and which intersects this line at the point $(1, 4)$ includes the point $(7, b)$ where $b = ?$

Answer: 16

Round 2

Division 3

Comprehensive A

Question 2

The intersection of a sphere of radius 10 with a plane having distance 7 from the center of the sphere is a circle of what area?

Answer: 51π

Round 2

Division 3

Comprehensive B

Question 1

If the polynomial

$$f(x) = x^2 - 13x + 10$$

is divided by the polynomial

$$g(x) = x - 2$$

what is the remainder?

Answer: -12

Round 2

Division 3

Comprehensive B

Question 2

What is the greatest value of c such that the system of equations

$$x^2 + y^2 = 25$$

$$x + y = c$$

has a (real) solution?

Answer: $5\sqrt{2}$

Round 2

Division 3

Team

Question 1

If

$$a_n = (n + 2)^2 - (n + 1)^2$$

find

$$a_1 + a_2 + a_3 + \dots + a_{2003}$$

Answer: 4020021

Round 2

Division 3

Team

Question 2

A parabola's focus F is the midpoint of a side of a square of area 1. Its directrix is the line containing the side of the square opposite the side containing the focus. The parabola intersects the square at points P and Q where $|PF| = |QF| = ?$

Answer: $\frac{5}{8}$

Round 3

Division 3

Geometry

Question 1

The base of a pyramid is a square measuring 4 meters by 4 meters. The height of the pyramid is 9 meters. What is the volume of the pyramid in cubic meters?

Answer: 48

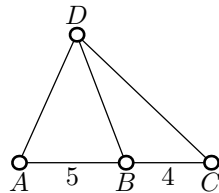
Round 3

Division 3

Geometry

Question 2

If $\triangle BCD$ has area 12, what is the area of $\triangle ABD$?



Answer: 15

Round 3

Division 3

Algebra

Question 1

$$\left(\frac{4^{-3}}{3^{-4}}\right)^{1/2} = ?$$

Answer: $\frac{9}{8}$

Round 3

Division 3

Algebra

Question 2

If $x \neq 0$ and $y \neq 0$ but

$$6x^2y^3 + 27xy^4 = 0$$

find

$$\frac{x}{y}$$

Answer: $-\frac{9}{2}$

Round 3

Division 3

Comprehensive A

Question 1

If $f(x) = x^2 + x$, what is $f(6) - f(3) \cdot f(2)$?

Answer: -30

Find the sum of the solutions of

$$\sin 2x + \cos x = 0$$

for

$$0 \leq x < 2\pi$$

Answer: 5π

If the side length of a square were doubled, the area of the square would increase by 9 square units. What is the side length of the square?

Answer: $\sqrt{3}$

Round 3

Division 3

Comprehensive B

Question 2

How many integer solutions does the equation

$$1 \leq |x + 2| \leq 4$$

have?

Answer: 8

Round 3

Division 3

Team

Question 1

Two sides of a triangle measure 1 and 7 and the angle included between them is 120° . What is the measure of the third side of the triangle?

Answer: $\sqrt{57}$

Round 3

Division 3

Team

Question 2

Two numbers are such that their difference, sum, and product are to each other as $1 : 7 : 36$ respectively. Find the sum of the two numbers.

Answer: 21

Round 4

Division 3

Geometry

Question 1

The angles of a triangle measure x° , $5x^\circ$, and $6x^\circ$. What is x ?

Answer: 15

Round 4

Division 3

Geometry

Question 2

Two solid iron spheres of radii 1 meter and 2 meters are melted down and the resulting iron used to cast a solid right circular cylinder with a height of 1 meter. What is the radius of the base of the cylinder in meters?

Answer: $2\sqrt{3}$

Round 4

Division 3

Algebra

Question 1

$$\log_{\sqrt{10}}(100\sqrt{10}) = ?$$

Answer: 5

Round 4

Division 3

Algebra

Question 2

Find x such that

$$\sum_{n=1}^{\infty} x^n = 5$$

Answer: $\frac{5}{6}$

Round 4

Division 3

Comprehensive A

Question 1

If

$$4x^2 + bx + 2 = 0$$

has exactly one real solution, find $|b|$.

Answer: $4\sqrt{2}$

Round 4

Division 3

Comprehensive A

Question 2

How many ways are there to choose 1 dessert and 3 side orders from a menu having 3 desserts and 10 side orders?

Answer: 360

Round 4

Division 3

Comprehensive B

Question 1

If

$$4y = x - 2$$

$$3x = 4 - y$$

then what is the value of y ?

Answer: $-\frac{2}{13}$

Round 4

Division 3

Comprehensive B

Question 2

Two lines each tangent to a circle of radius 2 intersect at right angles at a point P . What is the distance between P and the center of the circle?

Answer: $2\sqrt{2}$

Round 4

Division 3

Team

Question 1

Find

$$\sec(\tan^{-1}(-2))$$

Answer: $\sqrt{5}$

Round 4

Division 3

Team

Question 2

Find the minimum value of $x^4 - 6x^2 + 2$ for $-\infty < x < \infty$.

Answer: -7

Round 5

Division 3

Geometry

Question 1

The surface areas of two cubes are in a ratio of $4 : 9$. If V_1 is the volume of the smaller cube

and V_2 the volume of the larger cube, what is $\frac{V_1}{V_2}$?

Answer: $\frac{8}{27}$

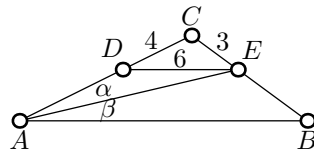
Round 5

Division 3

Geometry

Question 2

If $m \angle \alpha = m \angle \beta$ and $\overline{DE} \parallel \overline{AB}$, what is EB ?



Answer: $\frac{9}{2}$

Round 5

Division 3

Algebra

Question 1

Find a negative solution of

$$|22x + 1| = 19$$

Answer: $-\frac{10}{11}$

Round 5

Division 3

Algebra

Question 2

The quotient and difference of two numbers are both equal to 9. What is the larger of the two numbers?

Answer: $\frac{81}{8}$

Round 5

Division 3

Comprehensive A

Question 1

Solve for x :

$$27^{4x} = 9^{x+1}$$

Answer: $\frac{1}{5}$

Round 5

Division 3

Comprehensive A

Question 2

A diagonal of a face of a cube has length 4. What is the length of a diagonal of the cube?

Answer: $2\sqrt{6}$

Round 5

Division 3

Comprehensive B

Question 1

An equilateral triangle has area $2\sqrt{3}$. What is the length of a side?

Answer: $2\sqrt{2}$

Round 5

Division 3

Comprehensive B

Question 2

Find the sum of the digits of

$$10^{2003} - 10^{1003}$$

Answer: 9000

Round 5

Division 3

Team

Question 1

A sphere S is inscribed in a cube C . Find

$$\frac{\text{the surface area of } S}{\text{the surface area of } C}$$

Answer: $\frac{\pi}{6}$

Round 5

Division 3

Team

Question 2

If

$$xy = 6$$

$$xz = 8$$

$$yz = 12$$

find

$$x^2 + y^2 + z^2$$

Answer: 29