2008 ALABAMA

STATEWIDE MATHEMATICS CONTEST

First Round : March 29, 2008
Second Round: April 26, 2008 at The University of Alabama

DIVISION III COMPREHENSIVE EXAM

Construction of this test directed
by
Robert Moore, The University of Alabama

INSTRUCTIONS

This test consists of 50 multiple choice questions. The questions have not been arranged in order of difficulty. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one of more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

- All variables and constants, except those indicated otherwise, represent real numbers.
- Diagrams are not necessarily to scale.

We use the following geometric notation:

- If A and B are points, then:
  $\overline{AB}$ is the segment between A and B
  $\overleftrightarrow{AB}$ is the line containing A and B
  $\overrightarrow{AB}$ is the ray from A through B
  AB is the distance between A and B
- If A is an angle, then:
  $m\angle A$ is the measure of angle A in degrees
- If A and B are points on a circle, then:
  $\overset{\frown}{AB}$ is the arc between A and B
  $m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees

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What You Can Do With A Mathematics Major

Occupational opportunities

Actuarial and Insurance	Government	Accountant
Computer & Information Sciences	Investment Analyst	Financial Planner
Researcher	Benefits Specialist	Mathematician
Demographers	Computer Programmer	Cartographer
Data Processor	Navigator	Meteorologist
Applications Programmer	Ecologist	Health
Systems Analyst	Biomedical Engineer	Bio-mathematician
Computer Applications Engineer	Operations Analyst	Operations Research
Control Systems Engineer	Control Systems Engineer	Systems Engineer
Numerical Analyst	Teaching	BusinessIndustry
Statistician	Engineering Analyst	Financial Analyst
Technical Writer	Homeland Security	Communications Engineer

Study in the field of mathematics offers an education with an emphasis on careful problem analysis, precision of thought and expression, and the mathematical skills needed for work in many other areas. Many important problems in government, private industry, health and environmental fields, and the academic world require sophisticated mathematical techniques for their solution. The study of mathematics provides specific analytical and quantitative tools, as well as general problem-solving skills, for dealing with these problems. The University of Alabama offers undergraduate and graduates degrees in Mathematics. Please visit **www.ua.edu** and refer to the undergraduate and graduate programs for additional information.

Engineering Math Advancement Program

The University of Alabama is offering a new summer program to build math skills for students entering engineering. The Engineering Math Advancement Program (EMAP) is a summer residence class that addresses math and engineering prerequisites for incoming engineering students. The program targets bright students who may not have retained the information learned in high school and provides and opportunity to hone technical abilities before entering college. The goal of E-MAP is to assisting entering freshmen in developing a solid background in calculus to succeed in engineering before they start at the University.

Classes are designed around Precalculus Algebra and Trigonometry and incorporate important learning principles to ensure that knowledge is retained and not just memorized. Students develop their skills through hands-on experiences, problem solving teaming exercises, and interaction with engineering professors and instructors through an interdisciplinary Living Laboratory program. Experiments allow students to use simple calculus in engineering applications. The program also involves introducing students to local practicing engineers through work on one or more community service engineering-related activities. E-MAP will reserve 33-40 percent of enrollment space for underrepresented groups. Financial assistance is available based on need. Please visit **emap.ua.edu** for additional information.

1. Solve for x: $\quad 4^{x-1} = 8^{3x-1}$.

(**A**) $\frac{1}{6}$ (**B**) $\boxed{\frac{1}{7}}$ (**C**) 0 (**D**) -1 (**E**) None of these

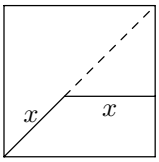
2. A cup contains 3 blue balls, 4 red balls and 1 white ball. 4 balls are chosen at random from the cup without replacement. What is the probability that *at least* one red ball is chosen?

(**A**) $\frac{59}{60}$ (**B**) $\frac{34}{35}$ (**C**) $\frac{1}{3}$ (**D**) $\frac{1}{2}$ (**E**) $\boxed{\text{none of these}}$

3. Solve for the positive root of the equation: $\quad (\sqrt{200}+\sqrt{56})\,x^2 + 10\,x - 2(\sqrt{50}-\sqrt{14}) = 0$.

(**A**) $\frac{\sqrt{26}}{\sqrt{14}}$ (**B**) $\sqrt{200}-\sqrt{56}$ (**C**) $\boxed{\frac{5\sqrt{2}-\sqrt{14}}{9}}$ (**D**) $\frac{10}{\sqrt{200}-\sqrt{56}}$ (**E**) none of these

4. The large square in the picture is 1 inch on a side. The point on the diagonal is equally distant from the bottom left corner and the right side. What is that distance (in inches)?



(**A**) $\boxed{2-\sqrt{2}}$ (**B**) $1+\sqrt{2}$ (**C**) $\sqrt{2}$ (**D**) $\sqrt{2}-1$ (**E**) None of these

5. Evaluate: $\quad \cos 67^\circ \cos 22^\circ + \cos 23^\circ \cos 68^\circ$.

(**A**) $\sqrt{2}$ (**B**) $\cos 89^\circ$ (**C**) 0 (**D**) $\boxed{1/\sqrt{2}}$ (**E**) none of these

6. How many ways are there to rearrange the letters of the word SUSURRUS?

(**A**) $8!$ (**B**) $\frac{8!}{5!\,3!}$ (**C**) $\boxed{\binom{8}{3}\binom{5}{3}}$ (**D**) $\binom{8}{5}$ (**E**) none of these

7. A large square is drawn, with area A. A circle is inscribed in the square and a smaller square is inscribed in the circle. What is the area of the smaller square?

(**A**) $\boxed{A/2}$ (**B**) $A/4$ (**C**) $(1/\sqrt{2})A$ (**D**) $\sqrt{A}$ (**E**) none of these

8. Town B is 3 miles west of town A. Town C is 2 miles north of town B. Town D is 3 miles northeast of town C. How far (in miles) is town D from town A?

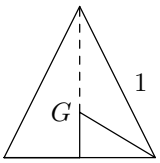
(**A**) $\sqrt{\frac{35}{2}-3\sqrt{2}}$ (**B**) $22-3\sqrt{2}$ (**C**) $\boxed{\sqrt{22-3\sqrt{2}}}$ (**D**) 8 (**E**) none of these

9. Evaluate the following, given that an answer exists and is less than e:

$$\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdots}}}$$

(**A**) 4 (**B**) $\sqrt{2}$ (**C**) $\boxed{2}$ (**D**) $2\sqrt{2}$ (**E**) none of these

10. Point G is the centroid of the equilateral triangle. What is the area of the small triangle at the lower right in the picture?



(**A**) $\sqrt{3}$ (**B**) $\sqrt{2}$ (**C**) $\frac{1}{2}\sqrt{3}$ (**D**) $\frac{1}{3}\sqrt{3}$ (**E**) none of these

11. Simplify: $\sqrt{28+\sqrt{300}}$.

(**A**) 5 (**B**) $25+\sqrt{3}$ (**C**) $5+\sqrt{3}$ (**D**) $5+2\sqrt{3}$ (**E**) none of these

12. In a large lecture class, 63 of the students are freshmen and 46 are math majors. 8 students are neither math majors nor freshmen. How many students are in the class?

(**A**) 109 (**B**) 117 (**C**) 101

(**D**) There is not enough information to solve the problem (**E**) none of these

13. On Tuesday, a merchant remarks all his prices, increasing them by 25%. On Wednesday he puts a sign in the window saying "SALE!!! Every item 25% off marked price!!!" What is the result of these changes?

I. Wednesday's prices are the same as Monday's.
II. Wednesday's prices are 25% less than Monday's .
III. Wednesday's prices are 93.75% of Monday's.
IV. Wednesday's prices are about 106.25% of Monday's

(**A**) I (**B**) II (**C**) III (**D**) IV (**E**) none of these

14. a is a real number larger than 1. Simplify the expression: $(2^{-\log_a(a^3)})\cdot(a^{-\log_a(1/2)})$.

(**A**) $\log_a 2$ (**B**) $\frac{1}{4}$ (**C**) $\frac{1}{2}$ (**D**) $2a$ (**E**) none of these

15. Solve the equation:

$$\begin{vmatrix} 1 & 1 \\ 2x & 3x \end{vmatrix} = x$$

where the vertical bars mean the determinant.

(**A**) $x=0$ (**B**) $x=1$ (**C**) $x=0$ or $x=1$

(**D**) Every value of x satisfies this equation (**E**) none of these

16. For real values of x, what is the smallest value assumed by the expression $(2x-1)(x+3)$?

(**A**) 0 (**B**) 1/2 (**C**) $-49/8$ (**D**) -3 (**E**) none of these

17. If x is very close to 1 — but not equal to 1 — then the expression

$$\frac{x^2-1}{x-1}$$

is very close to what?

(**A**) 0 (**B**) 1 (**C**) 2 (**D**) undefined (**E**) none of these

18. A plumber wants to replace an old 6-inch diameter pipe by some newer pipes, which are only 1 inch in diameter. How many of the new pipes will he need to get the same cross-sectional area?

(**A**) 6 (**B**) 6π (**C**) 36π (**D**) $\boxed{36}$ (**E**) none of these

19. A triangle has sides of length 24, 10, and 26. What is the radius of the inscribed circle?

(**A**) 26 (**B**) $\boxed{4}$ (**C**) 3 (**D**) 8 (**E**) none of these

20. The price of blue socks is twice the price of black socks. If John buys 20 pairs of black socks and 30 pairs of blue socks he pays \$120. How much does one pair of blue socks cost?

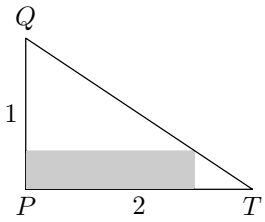
(**A**) \$1.50 (**B**) $\boxed{\$3.00}$ (**C**) \$4.50 (**D**) \$1.00 (**E**) none of these

21. The real roots of the polynomial $x(x^2 - 3x + 2)(x - 4)$ are

(**A**) 4 only (**B**) 0 and 4 (**C**) 0, 1, and 2 (**D**) $\boxed{0,\ 1,\ 2 \text{ and } 4}$ (**E**) none of these

22. The rectangle shown is inscribed in the right triangle with sides lengths 1 and 2. What is the largest possible area of the rectangle?

(**A**) 1 (**B**) $\boxed{1/2}$ (**C**) 1/3 (**D**) 1/4 (**E**) none of these



23. Solve for a in terms of b: $\frac{a}{b} = a - b$.

(**A**) $a = \dfrac{b}{b-1}$ (**B**) $\boxed{a = \dfrac{b^2}{b-1}}$ (**C**) $a = \dfrac{b}{b+1}$ (**D**) $a = \dfrac{b^2}{1-b}$ (**E**) none of these

24. The equation $x + \sqrt{x-2} = 4$ has:

(**A**) two real roots (**B**) one real and one purely imaginary root (**C**) $\boxed{\text{one root which is real}}$

(**D**) two complex roots, neither of which is purely imaginary (**E**) none of these

25. The front face of a rectangular box has area A_f. The area of the right side of the box is A_s and the area of the top is A_t. The product $A_f A_s A_t$ represents:

(**A**) the square of the surface area of the box (**B**) the total surface area of the box

(**C**) $\boxed{\text{the square of the volume of the box}}$ (**D**) the volume of the box (**E**) none of these

26. Line A is measured to be 10 inches long, with a possible relative error of $\pm 0.2\%$. Line B is measured to be 100 inches long, with a possible absolute error of ± 0.2 inches. Compared to the absolute error in the measurement of line A, the absolute error in line B is

(**A**) $\boxed{\text{greater by 0.18 inches}}$ (**B**) greater by 1.8 inches

(**C**) a hundred times greater (**D**) exactly the same (**E**) none of these

27. A regular hexagon is inscribed in a circle of radius r. What is the *perimeter* of the hexagon?

(**A**) $\boxed{6r}$ (**B**) $3r$ (**C**) $6\pi r$ (**D**) $3\pi r$ (**E**) none of these

28. If a and b are positive real numbers and if $a^{7/4} = b^{2/3}$, what is $\log_b a$?

(**A**) $21/8$ (**B**) $29/12$ (**C**) $13/12$ (**D**) $(8/21)\ln b$ (**E**) $\boxed{\text{none of these}}$

29. A fortnight is two weeks. A furlong is 660 feet, or 1/8 of a mile. If a turtle travels at 1 mile per hour, how fast is that in furlongs per fortnight?

(**A**) 42 (**B**) $1/42$ (**C**) $55/28$ (**D**) $\boxed{2688}$ (**E**) none of these

30. A convex polygon has 100 sides. How many different *diagonals* can be drawn in this figure?

(**A**) $\boxed{4850}$ (**B**) 4950 (**C**) 9900 (**D**) 8800 (**E**) none of these

31. Start with an equilateral triangle of side 3. Form a new triangle by joining the midpoints of the sides. Then form a third triangle by joining the midpoints of the sides of the second triangle. Continue in this fashion. The sum of the perimeters of all the triangles is:

(**A**) infinite (**B**) 6 (**C**) 13.5 (**D**) 15.75 (**E**) $\boxed{\text{none of these}}$

32. At 2 : 15 pm, the hands of a clock form an angle of:

(**A**) 30° (**B**) 5° (**C**) $\boxed{22.5^\circ}$ (**D**) 7.5° (**E**) none of these

33. Two cards are available. One card is red on both sides and one card is red on one side and blue on the other. One of the cards is chosen at random and placed on the table. The visible side is red. What is the probability that the other side is red?

(**A**) $1/2$ (**B**) $1/3$ (**C**) $\boxed{2/3}$ (**D**) $3/4$ (**E**) none of these

34. If $\frac{2}{3}$ of a can of paint will cover $\frac{3}{5}$ of a wall, how much of a can of paint is required to cover $\frac{3}{4}$ of the wall?

(**A**) $\boxed{\frac{5}{6}}$ (**B**) $\frac{6}{5}$ (**C**) $\frac{9}{30}$ (**D**) $\frac{10}{9}$ (**E**) none of these

35. If $i^2 = -1$, what is the value of $(1+i)^{16} - (1-i)^{16}$?

(**A**) $\boxed{0}$ (**B**) 1 (**C**) 2^8 (**D**) -2^8 (**E**) none of these

36. Simplify the expression $\dfrac{2(\sqrt{2}+\sqrt{6})}{3(\sqrt{2}+\sqrt{3})}$.

(**A**) $\frac{2\sqrt{2}}{3}$ (**B**) $\frac{2\sqrt{3}}{3}$ (**C**) $\frac{16}{9}$ (**D**) $\boxed{\frac{4}{3}}$ (**E**) none of these

37. Which is the largest of the following numbers?

(**A**) $\boxed{3^{6\cdot 9}}$ (**B**) $6^{3\cdot 9}$ (**C**) $9^{6\cdot 3}$ (**D**) $(3\cdot 6)^9$ (**E**) none of these

38. Three girls and three boys are seated around a circular table with six seats. They must alternate: boy, girl, boy, girl, etc. Ignoring rotations, how many different ways are there to seat the people? (Two configurations are different if there are two people who are neighbors in one configuration but not the other.)

(**A**) 3 (**B**) $\boxed{6}$ (**C**) 36 (**D**) 12 (**E**) none of these

39. Which of the following is equal to $\sqrt{4+2\sqrt{3}}$?

(**A**) $\sqrt{11}$ (**B**) $\boxed{1+\sqrt{3}}$ (**C**) $2+\sqrt{3}$ (**D**) $-1-\sqrt{3}$ (**E**) none of these

40. A boat in still water can travel at 3 miles per hour. But in a constant current, to travel 11 miles and return, the boat takes 24 hours. How fast is the current?

(**A**) 1 mph (**B**) $1\frac{1}{2}$ mph (**C**) 2 mph (**D**) 3 mph (**E**) $\boxed{\text{None of these}}$

41. In a race over d yards, A would beat B by 20 yards; B would beat C by 10 yards; and A would beat C by 28 yards. What is the value of d?

(**A**) $\boxed{100 \text{ yd}}$ (**B**) 58 yd (**C**) 116 yd

(**D**) not enough information to answer the question (**E**) none of these

42. a is a positive real number and e is the base for the natural logarithm. Simplify the expression: $\log_a(b^{e\cdot \ln a})$.

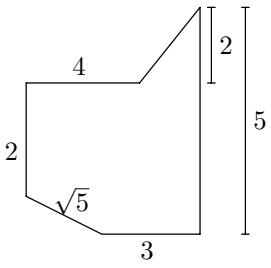
(**A**) $\dfrac{e}{\ln b}$ (**B**) $e\cdot\dfrac{\ln b}{\ln a}$ (**C**) $e\cdot\dfrac{\ln a}{\ln b}$ (**D**) $\left(\dfrac{\ln b}{\ln a}\right)^e$ (**E**) $\boxed{\text{none of these}}$

43. Five coins are flipped simultaneously. What is the probability that the number of heads showing is *even*?

(**A**) $\boxed{\dfrac{1}{2}}$ (**B**) $\dfrac{15}{32}$ (**C**) $\dfrac{14}{32}$ (**D**) $\dfrac{17}{32}$ (**E**) none of these

44. What is the area of the figure shown? (any angle that looks like a right angle really is one.)

(**A**) 14.5 (**B**) $\boxed{15.0}$ (**C**) 15.5 (**D**) $15-2\sqrt{5}$ (**E**) none of these



45. A right circular cone and a right circular cylinder have the same height and the same base. Compared to the cone, how large is the volume of the cylinder?

(**A**) 4 times as large as the volume of the cone (**B**) 2 times as large as the volume of the cone

(**C**) 3/2 times the volume of the cone (**D**) $\sqrt{3}$ times the volume of the cone (**E**) $\boxed{\text{none of these}}$

46. A three-person committee is to be chosen from a club of 15 people, and a chairperson of the committee will then be chosen. Two committees with the same membership but different chairs are considered different committees. How many different committees can be formed?

(**A**) 1200 (**B**) 455 (**C**) $\boxed{1365}$ (**D**) 1350 (**E**) none of these

47. We just blew some air into a spherical balloon and doubled its volume. By how much did we multiply the *surface area*?

(**A**) 2 (**B**) $2^{1/3}$ (**C**) $\boxed{2^{2/3}}$ (**D**) 2^3 (**E**) none of these

48. I have Q quarters and D dimes in my pocket, whose total value is \$2.10. Which of the following equations must be true?

(**A**) $Q + D = 210$ (**B**) $Q + D = 21$ (**C**) $10Q + 25D = 210$

(**D**) $25Q + 10D = 21$ (**E**) $\boxed{\text{none of these}}$

49. My mother's iced tea recipe uses a certain amount of sugar to make 1 gallon of tea. But I like it less sweet so I only use 4/5 of the sugar it calls for. Last night I made 3 and 3/4 gallons of tea. My sister is a sugar freak and uses 1 and 1/2 times as much sugar as the recipe calls for. Last night she made 2 and 1/4 gallons of tea. Who used more sugar?

(**A**) I used more sugar (**B**) $\boxed{\text{My sister used more sugar}}$ (**C**) We used the same amount of sugar

(**D**) The answer depends on the call of the original recipe (**E**) none of these

50. Which of the following numbers lies between $\frac{71}{140}$ and $\frac{37}{71}$?

(**A**) $\frac{70}{139}$ (**B**) $\frac{71}{144}$ (**C**) $\frac{1}{2}$ (**D**) $\boxed{\frac{72}{141}}$ (**E**) none of these