

INSTRUCTIONS

This test consists of 50 multiple choice questions. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a “perfect paper” with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

The questions have not been arranged in order of difficulty.

All variables and constants represent real numbers, except when a particular problem indicates otherwise.

Diagrams are not necessarily to scale.

We use the following geometric notation:

If A and B are points, then:

\overline{AB} is the segment between A and B

\overleftrightarrow{AB} is the line containing A and B

\overrightarrow{AB} is the ray from A through B

AB is the distance between A and B

If A is an angle, then:

$m\angle A$ is the measure of angle A in degrees

If A and B are points on a circle, then:

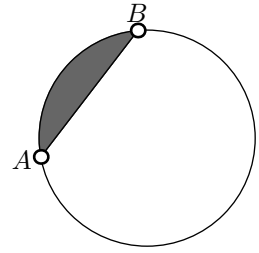
\widehat{AB} is the arc between A and B

$m\widehat{AB}$ is the measure of \widehat{AB} in degrees

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1. In the diagram, $AB = 6$ and $m\widehat{AB} = 90^\circ$. Find the area of the shaded region.

- (A) $36 + 18\pi$ (B) $\frac{9\pi}{2} - 9$ (C) $12 - 6\pi$
 (D) $18\pi - 36$ (E) Not enough information



2. Find an equation of the perpendicular bisector of the segment connecting $A = (-2, 3)$ and $B = (6, -5)$.

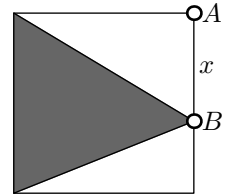
- (A) $x + 2y = -4$ (B) $2x + y = 2$ (C) $x - y = 3$ (D) $x + y = -1$ (E) $2x - 2y = -2$

3. An equilateral triangle has an altitude of length 6. Find its area.

- (A) 18 (B) $9\sqrt{2}$ (C) $12\sqrt{3}$ (D) $6\sqrt{3}$ (E) 10

4. The square has side length 4 and $AB = x$. Find the ratio of the shaded area to the unshaded area.

- (A) $\frac{1}{4}$ (B) $\frac{1}{4}$ (C) $\frac{4}{x}$ (D) $\frac{x+4}{x-4}$ (E) $4x$



5. In an isosceles trapezoid, two adjacent sides have lengths 5 and 12 respectively. Find the perimeter.

- (A) 22 (B) 27 (C) 34 (D) 40 (E) Not enough information

6. If V and A are the volume and surface area of the sphere of radius r , then:

- (A) $V = A$ (B) $VA = 9$ (C) $V - A = 6r$ (D) $3V = rA$ (E) $A = rV$

7. Three circles, each of radius 6, intersect in a way such that each circle passes through the centers of the other two circles. Find the area of the intersection of the three circles.

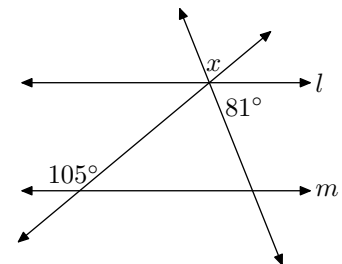
- (A) $18(\pi - \sqrt{3})$ (B) 9π (C) $6\sqrt{3} - \frac{\pi}{2}$ (D) 6π (E) $48\pi - \sqrt{3}$

8. Find the reflection of the point $(-5, 4)$ through the point $(2, -4)$.

- (A) $(9, -12)$ (B) $(4, -5)$ (C) $(-3, 0)$ (D) $(-7, 8)$ (E) $(-12, 12)$

9. In the diagram, $l \parallel m$. Find x .

- (A) 41° (B) 6° (C) 99° (D) 75° (E) 24°

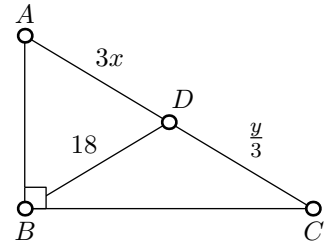


10. Find the volume of a cube whose longest diagonal has length 9 cm.

- (A) 27 cm^2 (B) $81\sqrt{3} \text{ cm}^2$ (C) $10\sqrt{2} \text{ cm}^2$ (D) 729 cm^2 (E) $9\sqrt{6} \text{ cm}^2$

11. In the right triangle shown, D is the midpoint of the hypotenuse. Find $x + y$.

- (A) 60 (B) 36 (C) 54 (D) 12 (E) Not enough information



12. Find the area of a triangle having sides of lengths 11 cm, 12 cm, and 13 cm.

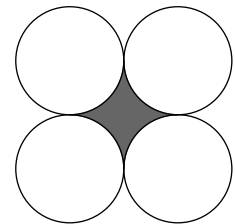
- (A) 143 cm^2 (B) 66 cm^2 (C) $6\sqrt{105} \text{ cm}^2$ (D) $10\sqrt{15} \text{ cm}^2$ (E) 36 cm^2

13. The sum of the lengths of the three medians of a triangle is:

- (A) Equal to the perimeter (B) Equal to one-half the perimeter (C) Greater than the perimeter
 (D) Less than the perimeter (E) Equal to one-third the perimeter

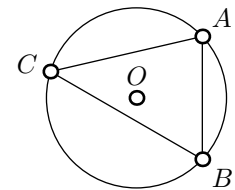
14. Four circles with 1 foot diameters are tangent as shown. Find the area of the shaded region in square feet.

- (A) $1 - \frac{\pi}{4}$ (B) $4\pi - 4$ (C) 4π (D) $\frac{1}{4} + \pi$ (E) $4 + 4\pi$



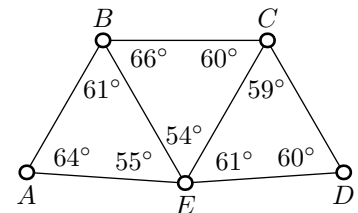
15. Find $m\angle ACB$ as shown in the circle of radius r and center O given that $AB = r$.

- (A) 15° (B) 90° (C) 45° (D) 60° (E) 30°



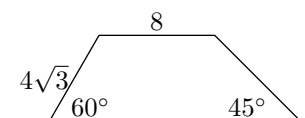
16. Given the diagram at right with the indicated angle measures, which segment is the longest?

- (A) \overline{AB} (B) \overline{CD} (C) \overline{AE} (D) \overline{CE} (E) \overline{BC}



17. Find the length of the larger diagonal in the given trapezoid.

- (A) $9\sqrt{27}$ (B) $32\sqrt{3}$ (C) $8\sqrt{2}$ (D) $2\sqrt{58}$ (E) $16\sqrt{6}$



18. The areas of two similar hexagons are 150 cm^2 and 200 cm^2 . What is the ratio of their perimeters?

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{\sqrt{2}}{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\sqrt{6}$ (E) $3\sqrt{2}$

19. A sphere of radius 5 meters and a right circular cone of radius 5 meters and height 10 meters rest on a flat table. With a knife traveling parallel to the table top, I wish to cut the tops off of both figures leaving horizontal cross-sections having equal areas. How far above the table should the cut be made?

(A) $2\sqrt{2} - 1$ m (B) $\frac{3}{2}$ m (C) $\sqrt{10}$ m (D) $\boxed{2 \text{ m}}$ (E) No such cut is possible.

20. Find the area of a rhombus having side length 12 and an interior angle of 150° .

(A) 48 (B) 60 (C) $\boxed{72}$ (D) 84 (E) 96

21. A rectangular table has legs of unequal length. Two legs on one diagonal of the table measure 24 and 28 inches and a third leg measures 30 inches. What should the fourth leg measure so that the table can rest on a level floor without wobbling?

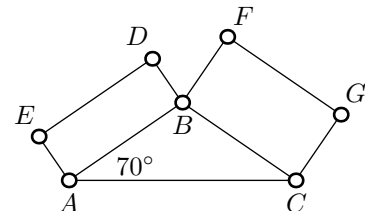
(A) 20 in (B) $\boxed{22 \text{ in}}$ (C) 24 in (D) 26 in (E) There is no such measure.

22. In $\triangle ABC$, $m\angle A = 30^\circ$, $m\angle C = 45^\circ$, and $AB = 8$. Find BC .

(A) $3\sqrt{7}$ (B) $2\sqrt{5}$ (C) $6\sqrt{3}$ (D) $\boxed{4\sqrt{2}}$ (E) $8\sqrt{11}$

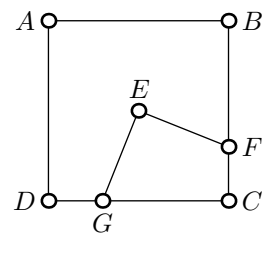
23. In the given figure, $AB = BC$, quadrilaterals $\square ABDE$ and $\square CBF G$ are both rectangles, and $m\angle BAC = 70^\circ$. What is $m\angle DBF$?

(A) 85° (B) 210° (C) $\boxed{140^\circ}$ (D) 90° (E) 15°



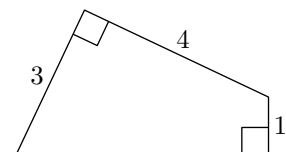
24. If $\square ABCD$ is a square with area 121 cm^2 , E is the center of the square, $\angle GEF$ is a right angle, and $FC = 3$. Find the area of $\square GEFC$.

(A) 18 cm^2 (B) 25.5 cm^2 (C) 81 cm^2
 (D) $62\frac{1}{8} \text{ cm}^2$ (E) $\boxed{30\frac{1}{4} \text{ cm}^2}$



25. Find the area of the given quadrilateral.

(A) 8 (B) $\boxed{6 + \sqrt{6}}$ (C) 12
 (D) $\sqrt{26}$ (E) Not enough information



26. A vertical 40 m pole and a vertical 60 m pole planted in a flat horizontal surface have bases which are 200 m apart. Find the height above the surface of the point of intersection of the lines joining the top of each pole to the bases of the other pole.

(A) $\boxed{24 \text{ m}}$ (B) 130 m (C) 100 m (D) 260 m (E) 50 m

27. A string is wound symmetrically around a right circular cylinder which has height 12 cm and circumference 6 cm, starting at one end of the cylinder and finishing at the other end, having gone exactly 6 times around the cylinder. How long is the string?

(A) 48 cm (B) $\boxed{12\sqrt{10} \text{ cm}}$ (C) $20\sqrt{3} \text{ cm}$ (D) $18\sqrt{5} \text{ cm}$ (E) 36 cm

28. Two circles of radius 1 lie in a plane and are tangent to each other. How many circles of radius 2 are in this same plane and tangent to both circles?

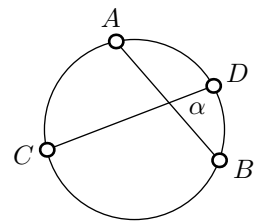
(A) 3 (B) 4 (C) $\boxed{5}$ (D) 6 (E) 7

29. Find the volume of a rectangular box whose top has area 12, front has area 16, and whose side has area 18.

(A) 72 (B) $8\sqrt{2}$ (C) $20\sqrt{3}$ (D) 40 (E) $\boxed{24\sqrt{6}}$

30. In the figure, $m\angle\alpha = 78^\circ$, and $m\widehat{AC} = 95^\circ$. Find $m\widehat{BD}$.

(A) 56° (B) 173° (C) $\boxed{61^\circ}$ (D) 112° (E) 17°

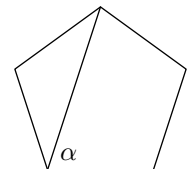


31. Find the area of the locus of points in the xy -plane whose distance from the circle with equation $x^2 + y^2 = 49$ is no more than 5.

(A) 196π (B) 189π (C) $\boxed{140\pi}$ (D) 95π (E) 45π

32. Find $m\angle\alpha$ as shown in the given regular pentagon.

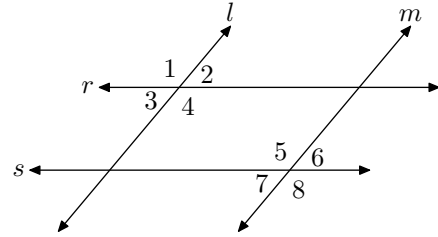
(A) 36° (B) 120° (C) 108° (D) $\boxed{72^\circ}$ (E) 60°



33. In $\triangle ABC$, $AB = 6$, $AC = 10$, and $BC = 8$. If D lies on \overline{BC} , and \overline{AD} bisects $\angle BAC$, then $BD = ?$

(A) 7 (B) 6 (C) 5 (D) 4 (E) $\boxed{3}$

34. In the figure, $l \parallel m$ and $r \parallel s$. If $m\angle 1 = 2x + 3y$, $m\angle 3 = x + y$, and $m\angle 5 = 2x + y + 30^\circ$, find x .
- (A) 20° (B) 30° (C) 40° (D) 50° (E) 60°



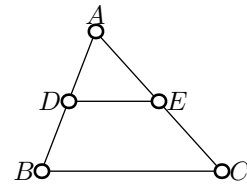
35. The sides of a triangle measure $x + 5$, $2x - 1$, and 4. For how many values of x will the triangle be isosceles?
- (A) 1 (B) 5 (C) 2 (D) 4 (E) 3

36. In $\triangle ABC$, $\overline{AB} \cong \overline{AC}$, and $m\angle B < 60^\circ$. Then:

- (A) \overline{BC} is the largest side of $\triangle ABC$ (B) \overline{BC} is the shortest side of $\triangle ABC$ (C) $\angle A$ is obtuse (D) $\triangle ABC$ is equilateral (E) Not enough information

37. In the figure below, D is the midpoint of \overline{AB} , $\overline{DE} \parallel \overline{BC}$, and the area of $\triangle ABC$ is 24m^2 . What is the area of $\triangle ADE$?

- (A) 6in^2 (B) 8in^2 (C) 10in^2 (D) 12in^2 (E) 14in^2

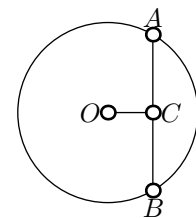


38. Find the area inside a regular hexagon of side length 8 cm.

- (A) 64cm^2 (B) $96\sqrt{3}\text{cm}^2$ (C) $16\sqrt{2}\text{cm}^2$ (D) 20cm^2 (E) $48\sqrt{5}\text{cm}^2$

39. In the given circle with center O , chord \overline{AB} has length 8, $OC = 10$, and $\overline{OC} \perp \overline{AB}$. Find the radius of the circle.

- (A) 2 (B) $6\sqrt{5}$ (C) 16 (D) $2\sqrt{29}$ (E) 80



40. If a circle of radius 10 cm has its radius decreased by 3 cm, by what percent is its area decreased?

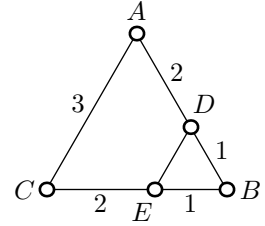
- (A) 30% (B) 70% (C) 49% (D) 51% (E) 35%

41. The exterior angles of an octagon add up to how many degrees?

- (A) 135° (B) 45° (C) 180° (D) 360° (E) 720°

42. In the figure $BD = BE = 1$ and $\triangle ABC$ is an equilateral triangle of side length 3. Find the perimeter of the quadrilateral $\square ADEC$.

(A) 6 (B) $6\frac{1}{2}$ (C) 7 (D) $7\frac{1}{2}$ (E) 8

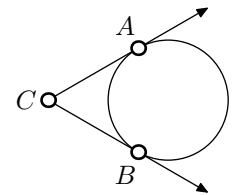


43. If $\square ABCD$ is a quadrilateral which is circumscribed about a circle and $AB = 4$, $BC = 5$, and $CD = 6$, then $AD =$:

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

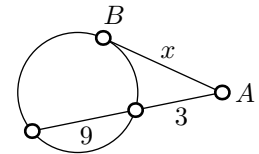
44. In the diagram, $m\widehat{AB} = 100^\circ$. Find the measure of $\angle ACB$.

(A) 50° (B) 60° (C) 70° (D) 80° (E) 90°



45. Find x given that \overline{AB} is tangent to the circle at B .

(A) 3 (B) 9 (C) $3\sqrt{3}$ (D) 12 (E) 6



46. The triangle with vertices $(1, 1)$, $(4, 5)$, and $(0, 2)$ is:

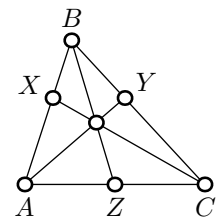
(A) A scalene triangle (B) An isosceles triangle (C) An equilateral triangle
(D) A right triangle (E) None of the above

47. The center of the circle circumscribed about a scalene triangle is the point of intersection of the:

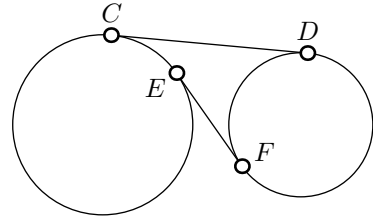
(A) Altitudes (B) Medians (C) Angle bisectors
(D) Perpendicular bisectors of the sides (E) None of the above

48. In the figure, $\frac{AX}{AB} = \frac{BY}{BC} = \frac{1}{3}$. What is $\frac{AZ}{AC}$?

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{6}$ (E) Not enough information

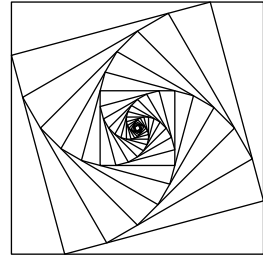


49. In the figure, segment \overline{CD} is tangent to both circles with points of contact at C and D . Also, segment \overline{EF} is tangent to both circles with points of contact at E and F . If $CD = 4$ and the large circle and small circle have radii of lengths 3 and 1, respectively, then find EF .



- (A) $3\sqrt{5}$ (B) $\frac{12}{5}$ (C) $3\sqrt{5} - 4$ (D) $\frac{3}{2}$ (E) 2

50. A square is drawn. Then the square is rotated 15° counterclockwise, and the new square is shrunk just enough so that its vertices lie on the sides of the original square. This process is repeated over and over forming the given figure. If S_1 denotes the original square and S_n denotes the n th square formed, what is the smallest value of n such that the area of the square S_n is less than one-tenth the area of the original square S_1 ?



- (A) 6 (B) 7 (C) 8 (D) 9 (E) 10