

GEOMETRY EXAMINATION

2002

ALABAMA STATE-WIDE MATHEMATICS CONTEST

Construction of this test directed

by

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INSTRUCTIONS

This test consists of 50 multiple choice questions. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a “perfect paper” with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one or more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

The questions have not been arranged in order of difficulty.

All variables and constants represent real numbers, except when a particular problem indicates otherwise.

Diagrams are not necessarily to scale.

We use the following geometric notation:

If A and B are points, then:

\overline{AB} is the segment between A and B

\overleftrightarrow{AB} is the line containing A and B

\overrightarrow{AB} is the ray from A through B

AB is the distance between A and B

If A is an angle, then:

$m\angle A$ is the measure of angle A in degrees

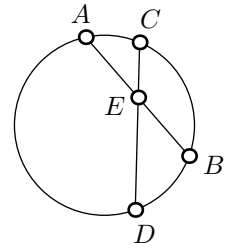
If A and B are points on a circle, then:

\widehat{AB} is the arc between A and B

$m\widehat{AB}$ is the measure of \widehat{AB} in degrees

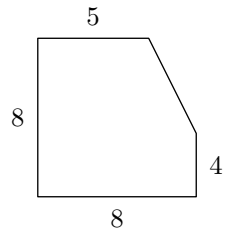
1. Given $AE = 6$, $EB = 5$, and $CE = 4$. Find DE .

(A) 7 (B) $7\frac{1}{2}$ (C) 8 (D) $4\frac{4}{5}$ (E) 9



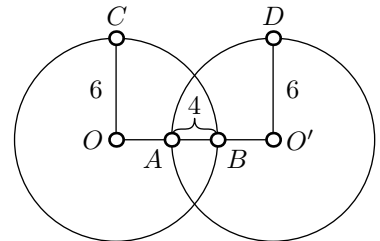
2. A square has a corner cut out as indicated. Find the area of the remaining figure.

(A) 64 (B) 40 (C) 58 (D) 44 (E) 72



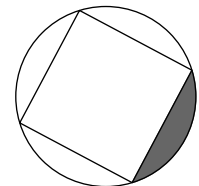
3. Congruent circles O and O' are drawn. The length of $\overline{OO'}$ is:

(A) 8 (B) 12 (C) 6 (D) 10 (E) 14



4. A square is inscribed in a circle of radius 6. The area of the shaded portion is:

(A) $36\pi - 18\sqrt{2}$ (B) $9\pi - 18$ (C) $12\pi - 4$ (D) $144 - 36\pi$ (E) $36 - 9\pi$



5. How many diagonals may be drawn in a 9-sided convex polygon?

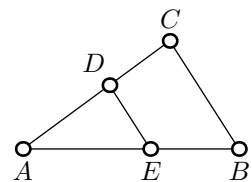
(A) 27 (B) 36 (C) 45 (D) 63 (E) 72

6. One interior angle of a regular polygon is 140° . How many sides does the polygon have?

(A) 9 (B) 6 (C) 12 (D) 18 (E) 40

7. Given that $\overline{DE} \parallel \overline{BC}$ and the area of $\triangle ADE$ is $\frac{1}{9}$ times the area of $\triangle ABC$. Then \overline{CB} is how many times the length of \overline{DE} ?

(A) 9 (B) 6 (C) 3 (D) 1 (E) 5

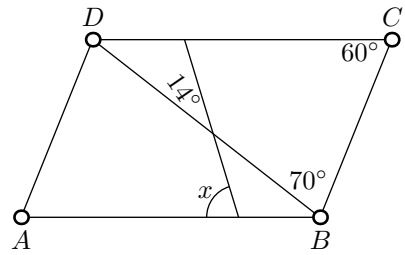


8. A person builds an enclosure with 120 feet of fencing. Which of the following shaped enclosures would have the maximum area?

(A) Triangle (B) Rectangle (C) Square (D) Regular Hexagon (E) Regular Pentagon

9. Figure $\square ABCD$ is a parallelogram. The number of degrees in $\angle x$ is:

(A) 64° (B) 70° (C) 60° (D) 76° (E) 90°

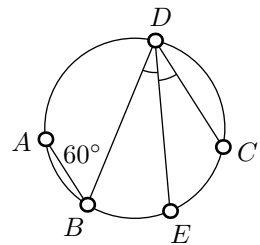


10. A conical sand pile has a circular base with a 12 foot diameter, and it is 4 feet high. How many cubic feet of sand are in the pile?

(A) 48π (B) 144π (C) 64π (D) 72π (E) 12π

11. In the figure $\overline{AB} \parallel \overline{CD}$, $\angle B = 60^\circ$, and \overline{DE} bisects $\angle D$. How many degrees are in arc \widehat{CE} ?

(A) 15° (B) 30° (C) 60° (D) 120° (E) 90°

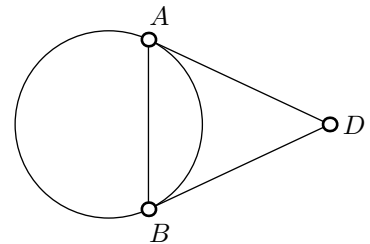


12. Approximately how many square feet of asbestos are needed to cover 100 feet of $3\frac{1}{2}$ inch diameter pipe?

(A) 3800 (B) 2200 (C) 1100 (D) 77 (E) 480

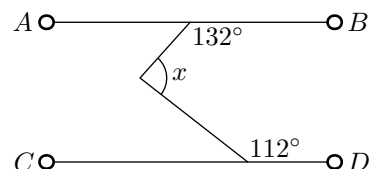
13. If \overline{AD} and \overline{BD} are tangent to circle O , and AD is twice AB . then $\triangle ADB$ is:

(A) Right (B) Isosceles (C) Equilateral
(D) Obtuse (E) Cannot be determined from the given information



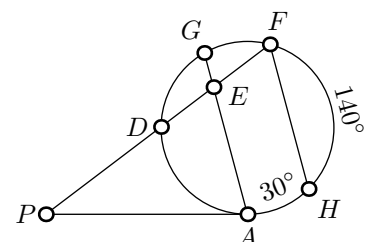
14. If $\overline{AB} \parallel \overline{CD}$, the number of degrees in $\angle x$ is:

(A) 48° (B) 116° (C) 68° (D) 64° (E) 140°



15. If \overline{PA} is tangent to the circle at the point A , \overline{PF} is a secant to the circle, $\widehat{DG} \cong \widehat{GF}$, $\overline{GA} \parallel \overline{FH}$, $m\widehat{FH} = 140^\circ$, and $m\widehat{HA} = 30^\circ$, then $m\angle P$ is:

(A) 15° (B) 40° (C) 30° (D) 20° (E) 60°

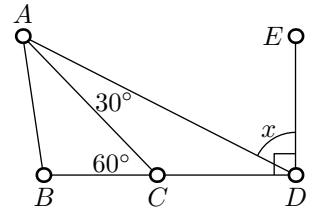


16. If $\frac{2}{9}$ of a side of a rhombus is 12, what is its perimeter?

- (A) 54 (B) 108 (C) 144 (D) 180 (E) 216

17. In the given figure, if $\overline{DE} \perp \overline{DB}$, then $m\angle x =$.

- (A) 15° (B) 30° (C) 45° (D) 60° (E) 75°



18. If the volume of a cube is 125, then the sum of the lengths of all the edges is:

- (A) 60 (B) 5 (C) 40 (D) 150 (E) 200

19. What is the least number of distinct lines in a plane that can be drawn so that the total number of points of intersection is 6?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

20. A square and an equilateral triangle have equal perimeters. If the area of the triangle is $16\sqrt{3}$, then the area of the square is:

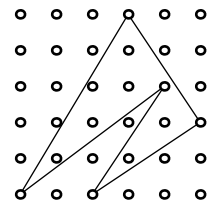
- (A) 9 (B) 16 (C) 25 (D) 36 (E) 576

21. A certain circle is given. The area of a regular hexagon which is inscribed in the circle is what percent of the area of a regular hexagon which is circumscribed about the circle?

- (A) 65% (B) 70% (C) 75% (D) 80% (E) Not enough information is given.

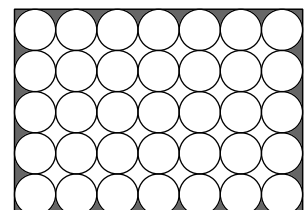
22. The vertical and horizontal distance between adjacent pairs of points is one. Find the area of the given five sided figure.

- (A) $\frac{15}{2}$ (B) 8 (C) $\frac{17}{2}$ (D) $\frac{19}{2}$ (E) 10



23. All the circles are tangent to one another and/or the sides of the rectangle. All the circles have radius 1. What is the area of the shaded region to the nearest whole unit, i.e., the region outside all the circles but inside the rectangle?

- (A) 7 (B) 8 (C) 9 (D) 10 (E) 11

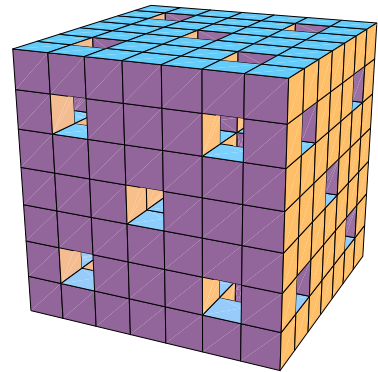


24. If the length and width of a certain rectangle are each increased by 1, the area of the new rectangle is 7 times the original area. If the length of the original rectangle is increased by 2 while the width is increased by 1 the area of the new rectangle is 10 times the original area. What is the perimeter of the original rectangle?

(A) $\frac{45}{2}$ (B) $\frac{9}{5}$ (C) 22 (D) $\frac{17}{3}$ (E) $\frac{10}{3}$

25. A cube is $7 \times 7 \times 7$. Through each face of the cube five unit square holes are drilled completely through the cube. Each face is drilled in the same way. What is the volume?

(A) 238 (B) 248 (C) 250 (D) 256 (E) 313

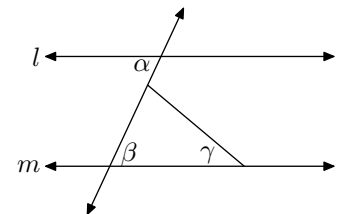


26. The degree measure of an interior angle of a regular hexagon is a . The degree measure of an exterior angle of a regular octagon is b . The ratio of a to b is:

(A) 3 : 8 (B) 4 : 3 (C) 2 : 1 (D) 8 : 3 (E) 3 : 1

27. If $\vec{l} \parallel \vec{m}$, $\angle \alpha = 65^\circ$, and $\angle \gamma = 45^\circ$, then $\angle \beta - \angle \gamma =$:

(A) 0° (B) 20° (C) 45° (D) 65° (E) 70°

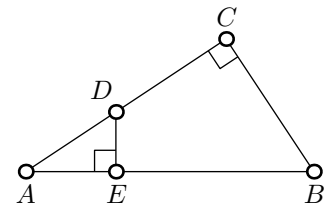


28. If the endpoints of a diameter of a circle are $(-3, -2)$ and $(11, -10)$, then the coordinates of the center of the circle are:

(A) $(2, -3)$ (B) $(4, -6)$ (C) $(8, -12)$ (D) $(-7, -6)$ (E) $(-4, -6)$

29. If $AD = 3$, $DE = 2$, and $AB = 15$, what is CB ?

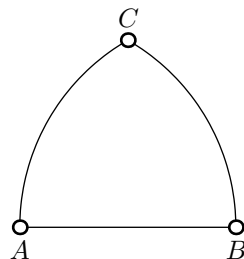
(A) $\sqrt{13}$ (B) $3\sqrt{13}$ (C) 10 (D) 26 (E) $2\sqrt{10}$



30. The area of a rhombus whose diagonals have lengths 6 and 9 is?

(A) 27 (B) 24 (C) $13\frac{1}{2}$ (D) 54 (E) 48

31. The gothic arch shown is constructed by using A as a center and AB as a radius to draw \widehat{CB} , and then repeating the process with B as center and AB as a radius to draw \widehat{CA} . If $AB = 4$, what is the area of the figure?

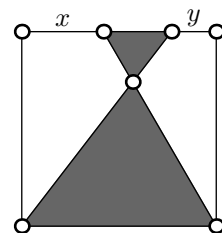


- (A) $\frac{16\pi - 12\sqrt{3}}{3}$ (B) $\frac{12\pi - 16\sqrt{3}}{3}$ (C) 16π
- (D) $\pi - 2$ (E) $\frac{32\pi - 8\sqrt{3}}{3}$
32. Given that w , x , y , and z are four distinct lines in a plane. If $w \perp x$, $y \parallel w$, and $z \parallel w$, then all of the following are false except:
- (A) $z \perp y$ (B) $z \parallel x$ (C) $w \perp z$ (D) $x \parallel y$ (E) $z \perp x$
33. If a rectangular prism has length 5, width 2, and height 4, then the total surface area is:
- (A) 40 (B) 56 (C) 66 (D) 68 (E) 76
34. If the edges of a cube are each doubled, what is the percent of increase in its volume?
- (A) 100% (B) 300% (C) 600% (D) 700% (E) 800%
35. Triangle $\triangle ABC$ has vertices $A = (-1, 2)$, $B = (3, 8)$, and $C = (5, -2)$. Find the area of the triangle $\triangle ABC$.
- (A) 12 (B) 26 (C) 42 (D) 56 (E) 66
36. A right circular cylinder whose radius is 6 inches and whose height is 10 inches is $\frac{2}{3}$ full of sand. How many cubic inches of sand are in the cylinder?
- (A) 40π (B) 80π (C) 240π (D) 120π (E) 360π
37. Which of the following is closest to the number of feet that an automobile tire of diameter 2 ft would roll in 350 revolutions of the tire?
- (A) 1500 ft (B) 1800 ft (C) 2200 ft (D) 3000 ft (E) 4200 ft
38. A tree 10 ft tall casts a shadow 25 ft long. At the same time a pole casts a shadow 5 ft long. The height of the pole in ft is?
- (A) 2 (B) $12\frac{1}{2}$ (C) 50 (D) 75 (E) 100

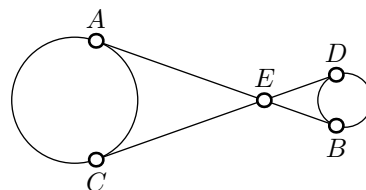
39. Given that the figure is a square with area 1, that $\frac{x}{y} = 7$, and that

$$\frac{\text{Area Large Shaded Triangle}}{\text{Area Small Shaded Triangle}} = 9. \text{ Find the shaded area.}$$

- (A) $\frac{7}{12}$ (B) $\frac{7}{9}$ (C) $\frac{7}{18}$ (D) $\frac{5}{14}$ (E) $\frac{5}{12}$



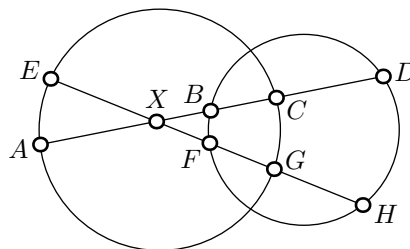
40. Given that \overline{AB} is tangent to the large and small circle at A and B , respectively, and that \overline{CD} is tangent to the large and small circle at C and D , respectively, and that $AC = 8$, $AE = 6$, and $ED = 1$, find the difference of the radii of the two circles.



- (A) $2\sqrt{2}$ (B) $3\sqrt{2}$ (C) $5\sqrt{2}$ (D) $2\sqrt{5}$ (E) $3\sqrt{5}$

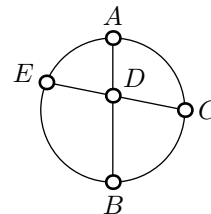
41. Given that $AX = 5$, $BC = 2$, $CD = 4$, $EX = 5$, $XF = 1$, and $GH = 3$, find $FG - XB$.

- (A) $\frac{3}{4}$ (B) $\frac{6}{5}$ (C) $\frac{4}{5}$ (D) $\frac{2}{3}$ (E) 1



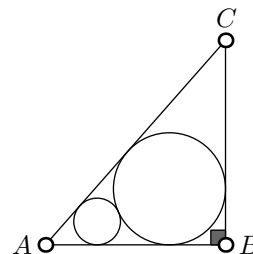
42. Given that $AD = 2$, $DB = 4$, $\widehat{AC} \cong \widehat{CB}$, and that \overline{AB} is a diameter of the circle, find EA .

- (A) $\frac{6\sqrt{5}}{5}$ (B) $2\sqrt{2}$ (C) $\frac{3\sqrt{2}}{2}$ (D) $\frac{3\sqrt{5}}{5}$ (E) $\frac{6\sqrt{10}}{5}$

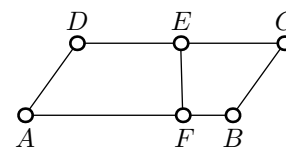


43. The circles are tangent to one another and each circle is tangent to the sides of the right triangle $\triangle ABC$ with right angle $\angle ABC$. If the larger circle has radius 12 and the smaller circle has radius 3, what is the area of the triangle?

- (A) 420 (B) 620 (C) 1240 (D) 1344 (E) 2688

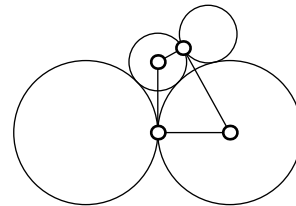


44. The figure $\square ABCD$ is a parallelogram, $\overline{DE} \cong \overline{EC}$, and $F \neq B$. If the segment \overline{EF} divides the area of the parallelogram in the ratio $p : 1$ where $p > 1$, then $\frac{AF}{FB} = ?$



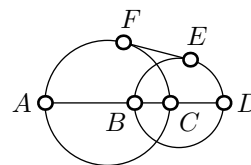
- (A) $\frac{3p-1}{3-p}$ (B) p^2 (C) $\frac{3p-1}{p+1}$ (D) $\frac{p+1}{2p}$ (E) $\frac{p+1}{p^2+1}$

45. All the circles which appear to be tangent in the diagram are tangent. Each of the two larger circles has a radius of 4 while each of the two smaller circles have a radius of 1. The quadrilateral is drawn from the center of the smaller circle to the point of tangency of the smaller circles to the center of the larger circle then to the point of tangency of the larger circles then back to the center of the smaller circle. Find the area of the quadrilateral.



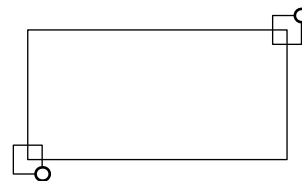
- (A) $6 + \sqrt{6}$ (B) $6\sqrt{6}$ (C) $4 + 2\sqrt{2}$ (D) $6 + \sqrt{10}$ (E) $5 + \sqrt{2}$

46. Segment \overline{EF} is tangent to the small and large circle at the points E and F , respectively. Segments \overline{AC} and \overline{BD} are diameters of the large and small circles, respectively. If $EF = 4$, and the radii of the large and small circles are 5 and 3, respectively, then $BC = ?$



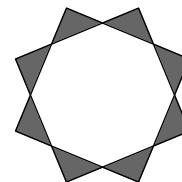
- (A) $3 + 2\sqrt{5}$ (B) $5 - 2\sqrt{2}$ (C) $5 - 2\sqrt{5}$ (D) $8 + 2\sqrt{5}$ (E) $8 - 2\sqrt{5}$

47. The large figure is a rectangle which is 3 by 5. At opposite corners of the rectangle are squares whose centers are vertices of the rectangle, and whose sides are parallel to the sides of the rectangle, and whose side lengths are each 1. Find the distance between the indicated vertices of the smaller squares.



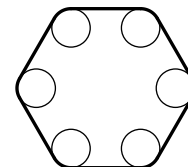
- (A) $3\sqrt{5}$ (B) $\sqrt{41}$ (C) $\sqrt{34}$ (D) $2\sqrt{10}$ (E) $2\sqrt{13}$

48. The inside of the given shaded figure is a regular octagon which could be inscribed in a circle of radius one. Find the shaded area.



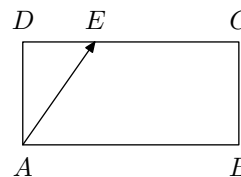
- (A) $3 - \sqrt{2}$ (B) $\frac{1 + \sqrt{2}}{2}$ (C) $\frac{2 + \sqrt{2}}{2}$ (D) $4 - 2\sqrt{2}$ (E) $\frac{4}{3}$

49. A belt passes over six wheels. The centers of the wheels form a regular hexagon of diameter six. Each wheel has diameter one. What is the length of the belt to the nearest whole number?



- (A) 42 (B) 20 (C) 22 (D) 35 (E) 21

50. A billiard table (no pockets) is 5 ft by 10 ft. A ball is shot from point A (the lower left corner) to point E . The ball then begins bouncing from side to side. After a certain number of bounces the ball will first strike side AD at some point F . What is the length of AF in feet, if $DE = 3$ ft? (Assume the balls are represented by points.)



- (A) 2 (B) $\frac{5}{3}$ (C) $\frac{10}{3}$ (D) 3 (E) $\frac{5}{2}$