GEOMETRY EXAMINATION

2003

ALABAMA STATE-WIDE MATHEMATICS CONTEST

Construction of this test directed

by

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This test consists of 50 multiple choice questions. For each question, choose the best of the five answer choices labeled A, B, C, D, and E.

The test will be scored as follows: 5 points for each correct answer, 1 point for each question left unanswered, and 0 points for each wrong answer. (Thus a "perfect paper" with all questions answered correctly earns a score of 250, a blank paper earns a score of 50, and a paper with all questions answered incorrectly earns a score of 0.)

Random guessing will not, on average, either increase or decrease your score. However, if you can eliminate one of more of the answer choices as wrong, then it is to your advantage to guess among the remaining choices.

The questions have not been arranged in order of difficulty.

All variables and constants represent real numbers, except when a particular problem indicates otherwise.

Diagrams are not necessarily to scale.

We use the following geometric notation:

If A and B are points, then:

$\overline{AB}$ is the segment between A and B

$\overleftrightarrow{AB}$ is the line containing A and B

$\overrightarrow{AB}$ is the ray from A through B

AB is the distance between A and B

If A is an angle, then:

$m\angle A$ is the measure of angle A in degrees

If A and B are points on a circle, then:

$\overset{\frown}{AB}$ is the arc between A and B

$m\overset{\frown}{AB}$ is the measure of $\overset{\frown}{AB}$ in degrees

Editing by Jeff Dodd, Jaedeok Kim, Youngmi Kim, and Steve White, JSU
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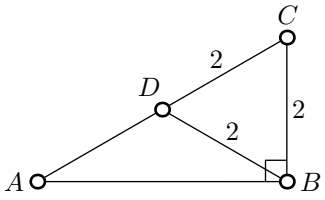
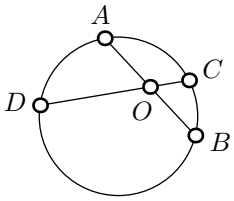
1. Each edge of $\triangle BCD$ has length 2, D lies on $\overline{AC}$, and $m\angle ABC = 90°$. What is the length of $\overline{AB}$?



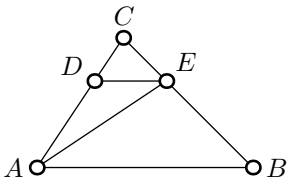
(**A**) 2 (**B**) 4 (**C**) $3\sqrt{2}$ (**D**) $\sqrt{3}$ (**E**) $2\sqrt{3}$

2. If $CO = 2$, $AB = 8$, and $OA = OB$. Find CD.



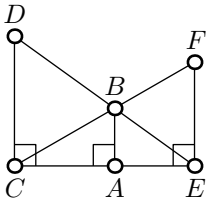
(**A**) $\sqrt{38}$ (**B**) 8 (**C**) 10 (**D**) $2\sqrt{5}$ (**E**) 32

3. Given that $\overline{AB} \parallel \overline{DE}$ and $DE : AB = 1 : 3$. If the area of triangle $\triangle CDE$ is 20, then the area of triangle $\triangle DEA$ is:



(**A**) 20 (**B**) 40 (**C**) 80 (**D**) 100 (**E**) 120

4. What is the length of AB, given that $CD = 24$ and $EF = 18$?

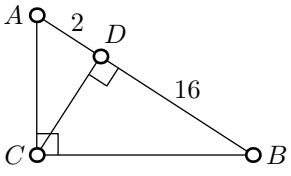


(**A**) $\frac{24}{7}$ (**B**) $\frac{36}{7}$ (**C**) $\frac{52}{7}$ (**D**) $\frac{72}{7}$ (**E**) $\frac{108}{7}$

5. What is the volume of a sphere of radius 6?

(**A**) 48π (**B**) 72π (**C**) 144π (**D**) 288π (**E**) 864π

6. In the right triangle $\triangle ABC$, the altitude from vertex C divides the hypotenuse into two segments, one of length 2 and the other of length 16. Find the perimeter of triangle $\triangle ABC$.



(**A**) $24 + 4\sqrt{14}$ (**B**) $18 + 2\sqrt{7} + 12\sqrt{2}$ (**C**) 36

(**D**) $24 + 12\sqrt{2}$ (**E**) $16 + 2\sqrt{14}$

7. What is the maximum number of points of intersection of two different lines and three different circles in the same plane?

(**A**) 9 (**B**) 13 (**C**) 19 (**D**) 22 (**E**) 25

8. A 5 inch by 8 inch rectangle is enlarged to a similar rectangle whose smallest side measures 9 inches. What is the length in inches of the diagonal of the enlarged rectangle?

(**A**) $\frac{5\sqrt{89}}{9}$ (**B**) $\frac{9\sqrt{33}}{5}$ (**C**) $\frac{5\sqrt{69}}{9}$ (**D**) $\frac{9\sqrt{89}}{5}$ (**E**) $\frac{9\sqrt{69}}{5}$

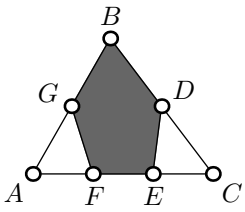
9. A bicycle has a 100 cm diameter wheel. If you ride on and around a circle with a 10 km diameter 12 times, how many revolutions does the wheel make?

(**A**) 12×10^2 (**B**) 12×10^3 (**C**) 12×10^4 (**D**) 12×10^5 (**E**) 12×10^6

10. A circle of radius 10 has its radius reduced by 4. By what percentage has its area been decreased?

(**A**) 36% (**B**) 40% (**C**) 64% (**D**) 70% (**E**) 80%

11. In triangle $\triangle ABC$, D bisects side $\overline{BC}$, G bisects side $\overline{AB}$, and the points E and F trisect side $\overline{AC}$. What is the area of the shaded polygon, if the area of $\triangle ABC$ is 108?

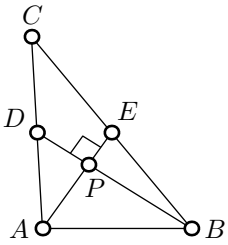


(**A**) 18 (**B**) 36 (**C**) 64 (**D**) 72 (**E**) 90

12. What is the area of the triangle whose vertices are $(0, 0)$, $(14, 6)$, and $(12, 9)$?

(**A**) 9 (**B**) 18 (**C**) 27 (**D**) 54 (**E**) 84

13. In a triangle $\triangle ABC$, $AC = 36$, $BC = 48$, and the medians $\overline{BD}$ and $\overline{AE}$ to sides $\overline{AC}$ and $\overline{BC}$, respectively, are perpendicular. Find AB.



(**A**) $12\sqrt{5}$ (**B**) $15\sqrt{2}$ (**C**) $6\sqrt{5}$ (**D**) $5\sqrt{6}$ (**E**) $5\sqrt{12}$

14. What is the angle between the hour hand and minute hand of a clock face at 2 : 25?

(**A**) 25° (**B**) 60° (**C**) $72\frac{1}{2}^\circ$ (**D**) $77\frac{1}{2}^\circ$ (**E**) 80°

15. To construct a circle that circumscribes a triangle one finds its center by locating the intersection of which two lines?

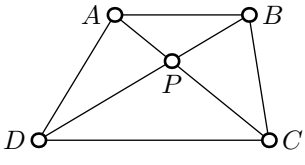
(**A**) Two medians (**B**) Perpendicular bisectors of two sides (**C**) Bisectors of two angles

(**D**) Two altitudes (**E**) None of the above

16. An equilateral triangle is inscribed in a circle. Each side of the triangle has length x. What is the area of the circle?

(**A**) $\frac{\pi x^2}{2}$ (**B**) $\frac{3\pi x^2}{4}$ (**C**) $\frac{\pi x^2}{4}$ (**D**) $\frac{\pi x^2}{3}$ (**E**) πx^2

17. Given $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, $AB = 9$, and $CD = 12$. If the area of triangle $\triangle CPD$ is 64, then the area of triangle $\triangle APB$ is?



(**A**) 9 (**B**) 12 (**C**) 18 (**D**) 24 (**E**) 36

18. Three circles are mutually tangent externally. Their centers form a triangle whose sides are of lengths 8, 9, and 13. Find the total area of the three circles.

(**A**) 64π (**B**) 89π (**C**) 108π (**D**) 229π (**E**) 314π

19. The length of a certain rectangle is quadrupled and the width is tripled. What is the ratio of the area of the new enlarged rectangle to the original rectangle?

(**A**) 3 : 1 (**B**) 7 : 2 (**C**) 4 : 1 (**D**) 5 : 2 (**E**) 12 : 1

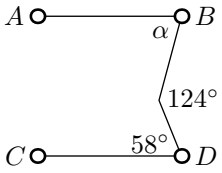
20. An equilateral triangle and a regular hexagon have equal perimeters. What is the area of the triangle, if the area of the hexagon is 120?

(**A**) 40 (**B**) 60 (**C**) 80 (**D**) 90 (**E**) 120

21. The larger angles of a rhombus are twice the smaller angle of the rhombus. If the shorter diagonal is 20, find the perimeter of the rhombus.

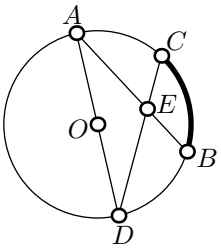
(**A**) 40 (**B**) $40\sqrt{3}$ (**C**) 60 (**D**) 80 (**E**) $80\sqrt{3}$

22. Given that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, find the number of degrees in angle $\angle\alpha$.



(**A**) 12° (**B**) 56° (**C**) 58° (**D**) 66° (**E**) 72°

23. Given that angle $m\angle A = 30°$, $m\angle D = 28°$, and $\overline{AD}$ is a diameter of the circle, find the number of degrees in arc $\widehat{BC}$.

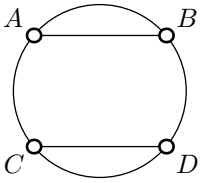


(**A**) 28° (**B**) 30° (**C**) 58° (**D**) 64° (**E**) 128°

24. Given that $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$, which of the following is true?

I. $m\,\widehat{AC} = m\,\widehat{BD}$ II. $m\,\widehat{AB} = m\,\widehat{CD}$

III. $m\,\widehat{AB} = m\,\widehat{BD}$ IV. $m\,\widehat{AB} = m\,\widehat{AC}$



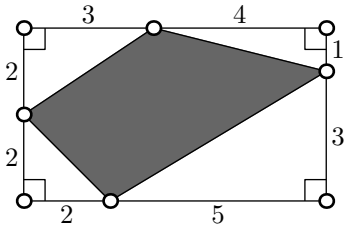
(**A**) I only (**B**) II only (**C**) I and IV only

(**D**) None of them (**E**) All of them

25. What is the number of different squares which can be inscribed in a given equilateral triangle?

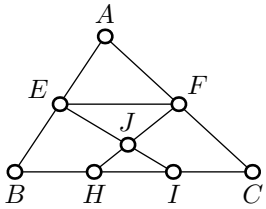
(**A**) 0 (**B**) 1 (**C**) 2 (**D**) 3 (**E**) 4

26. What is the area of the shaded portion of the figure?



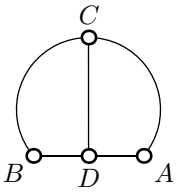
(**A**) $13\frac{1}{2}$ (**B**) $14\frac{1}{2}$ (**C**) $19\frac{1}{2}$ (**D**) 20 (**E**) 39

27. In triangle $\triangle ABC$, E and F are midpoints of sides $\overline{AB}$ and $\overline{AC}$, respectively; and H and I trisect the side $\overline{BC}$. If the area of triangle $\triangle ABC$ is 120, what is the area of triangle $\triangle EFJ$?



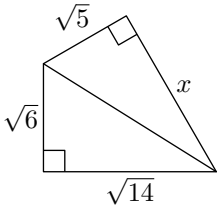
(**A**) 9 (**B**) 18 (**C**) 24 (**D**) 30 (**E**) 40

28. In the figure, $A\widehat{C}B$ is an arc of a circle, and $\overline{CD}$ is the perpendicular bisector of chord $\overline{AB}$. If $CD = 18$ and $AB = 12$, find the area of the entire circle.



(**A**) 36π (**B**) 81π (**C**) 100π (**D**) 144π (**E**) 324π

29. What is the value of x?

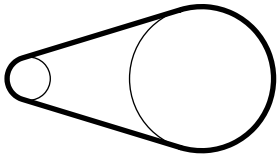


(**A**) $\sqrt{15}$ (**B**) 16 (**C**) $16\frac{4}{5}$ (**D**) $\sqrt{12}$ (**E**) $2\sqrt{5}$

30. Find the perimeter in inches of a regular polygon whose area is $24\sqrt{3}$ inches, and whose apothem, the perpendicular distance from the center of the regular polygon to one of its sides, is $2\sqrt{3}$ inches.

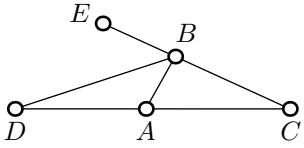
(**A**) 6 (**B**) $12\sqrt{3}$ (**C**) 18 (**D**) 24 (**E**) $24\sqrt{3}$

31. The radii of two pulley wheels are 5 inches and 1 inch. The distance between their centers is 8 inches. Find the length of a pulley belt which passes around the two pulley wheels.



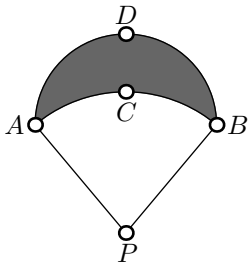
(**A**) $\frac{20\pi}{3}$ (**B**) $\frac{22\pi}{3} + 8\sqrt{3}$ (**C**) $\frac{28\pi}{3}$ (**D**) $\frac{20\pi}{3} + 4\sqrt{3}$ (**E**) $\frac{40\pi}{3}$

32. In the given figure, $\overline{DB}$ bisects the exterior angle $\angle EBA$ of triangle $\triangle ABC$. If $AB = 6$, $BC = 10$, and $AC = 12$, find DA.



(**A**) 12 (**B**) 14 (**C**) 16 (**D**) 18 (**E**) 20

33. The shaded crescent area is bounded by the semicircle $A\widehat{D}B$ with diameter $\overline{AB}$ and by the arc $A\widehat{C}B$ of a circle with center P. If $AB = 12$ and $AP = 12$, what is the area in inches of the shaded crescent area?

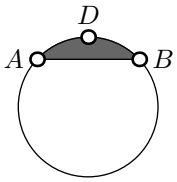


(**A**) $36\sqrt{3} - 6\pi$ (**B**) $18\sqrt{3} - 4\pi$ (**C**) 22π (**D**) $36\sqrt{3}\pi$ (**E**) 40π

34. How many sides has a regular polygon each interior angle of which measures $160°$?

(**A**) 12 (**B**) 18 (**C**) 20 (**D**) 24 (**E**) 36

35. Find the perimeter of the shaded area $ADBA$, if the radius of the circle is 12 and $m\, A\widehat{D}B = 120°$.



(**A**) 12π (**B**) $4\pi + 12\sqrt{3}$ (**C**) 18π (**D**) $8\pi + 6\sqrt{3}$ (**E**) $8\pi + 12\sqrt{3}$

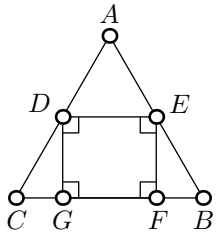
36. The vertices of a triangle $\triangle ABC$ are $A = (-3, 2)$, $B = (6, 5)$, and $C = (9, -4)$. Find the length of the median from B to side $\overline{AC}$.

(**A**) $2\sqrt{5}$ (**B**) 3 (**C**) $3\sqrt{5}$ (**D**) 4 (**E**) $5\sqrt{5}$

37. The sum of the areas of two regular decagons is 39 square inches, and their radii are in the ratio 2 : 3. Find the area of the larger decagon.

(**A**) 12 (**B**) 17 (**C**) 22 (**D**) 27 (**E**) 29

38. In the figure, triangle $\triangle ABC$ is an equilateral triangle. The points D and E are the midpoints of sides $\overline{AC}$ and $\overline{AB}$, respectively. What is the ratio of the area of the quadrilateral $\square DEFG$ to the area of triangle $\triangle ABC$?



(**A**) $\frac{1}{3}$ (**B**) $\frac{1}{2}$ (**C**) $\frac{5}{8}$ (**D**) $\frac{3}{4}$ (**E**) 1

39. Find an equation of the perpendicular bisector of the segment joining $(3, -1)$ and $(-1, 7)$.

(**A**) $x - 2y = -5$ (**B**) $x + 2y = 7$ (**C**) $2x + y = -5$ (**D**) $2x + y = 5$ (**E**) $2x - y = -1$

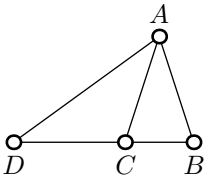
40. What is the volume of a regular square pyramid with base edge 16 cm and height 6 cm?

(**A**) 144 cm^3 (**B**) 216 cm^3 (**C**) 336 cm^3 (**D**) 512 cm^3 (**E**) 625 cm^3

41. A car travels North for 4 miles, then West for 6 miles, and then South West for 2 miles. How many miles is the car from its starting point?

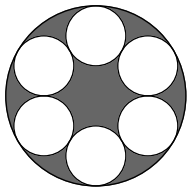
(**A**) $\sqrt{56 + 4\sqrt{2}}$ (**B**) $\sqrt{16 + 2\sqrt{2}}$ (**C**) $\sqrt{36 + 4\sqrt{2}}$ (**D**) $\sqrt{6 + 2\sqrt{2}}$ (**E**) $\sqrt{12 + 4\sqrt{2}}$

42. In the figure $AB = AC = CD$, and $AD = BD$. Find the measure of $\angle ADC$ in degrees.



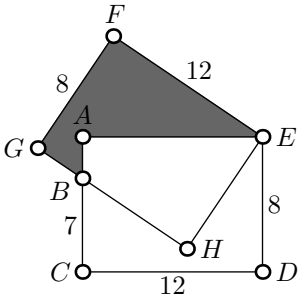
(**A**) 28° (**B**) 32° (**C**) 36° (**D**) 54° (**E**) 72°

43. Given six congruent circles drawn internally tangent to a circle of radius 18; each smaller circle is also tangent to each of its adjacent circles. Find the shaded area between the large circle and the six smaller circles.



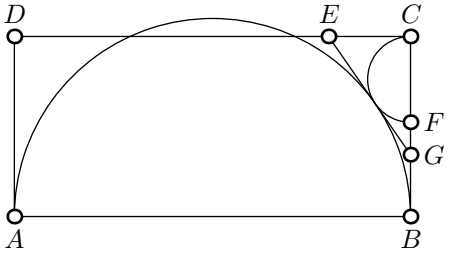
(**A**) 36π (**B**) 96π (**C**) 108π (**D**) 144π (**E**) 216π

44. In the given figure, the two rectangles $\square EFGH$ and $\square ACDE$ share a common corner at E and overlap so that $BC = 7$. What is the area of the shaded region $\square ABGFEA$?



(**A**) 36 (**B**) 54 (**C**) 64 (**D**) 72 (**E**) 96

45. The figure $\square ABCD$ is a rectangle, $AD = 6$, $AB = 18$, arc $\overset{\frown}{AB}$ is a semicircle with diameter $\overline{AB}$, arc $\overset{\frown}{CF}$ is a semicircle with diameter $\overline{CF}$, and $\overline{EG}$ is tangent to both semicircles. What does $EC =$?

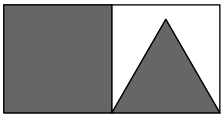


(**A**) $\frac{1}{2}$ (**B**) $\frac{2}{3}$ (**C**) $\frac{3}{4}$ (**D**) 1 (**E**) 2

46. If one were to cut five circles, each 2 cm in diameter, from a rectangular piece of paper 6 cm long, at least how many cm's wide would the piece of paper have to be?

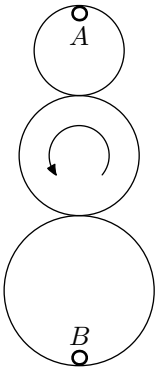
(**A**) $\sqrt{3}$ (**B**) $2 + \sqrt{3}$ (**C**) 4 (**D**) $3\sqrt{3}$ (**E**) 2

47. A square of side length S and an equilateral triangle of side length S are placed inside a rectangle of length $2S$ and width S as shown. What fraction of the area of the rectangle remains uncovered?



(**A**) $\frac{4+\sqrt{3}}{8}$ (**B**) $\frac{1+2\sqrt{3}}{2}$ (**C**) $\frac{4-\sqrt{3}}{8}$ (**D**) $\frac{1-2\sqrt{3}}{2}$ (**E**) $\frac{2+10\sqrt{3}}{4}$

48. Three friction gears are shown at left. They turn without any slippage and the centers of the gears are in line with one another. The diameters of the friction gears are, 6, 8, and 10, respectively. As the center friction gear rotates counter clockwise, it causes the top and bottom friction gear to rotate clockwise. The top and bottom friction gears each have a timing mark stamped on the friction gear, which of course rotate as the gear rotates. Timing mark A is initially at top dead center (at the very top of the friction gear), while timing mark B is at bottom dead center (at the very bottom of the friction gear). What is the minimum number of degrees that the middle friction gear must rotate in order for timing mark A to be at bottom dead center while timing mark B will be at top dead center?



(**A**) 180° (**B**) 900° (**C**) 405° (**D**) 540° (**E**) 675°

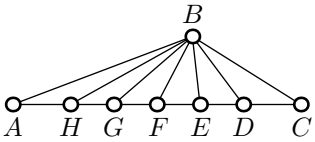
49. Given a pair of different lines which intersect one another, and a circle which does not intersect either line; how many different circles can be drawn which are tangent to both lines, and also tangent to the given circle?

(**A**) none (**B**) always exactly one (**C**) always exactly two

(**D**) either none or exactly one depending on the position of the circle and the lines

(**E**) either exactly one or exactly two depending on the position of the circle and the lines

50. In the given figure, $AB = 6$, $BC = 4$, $CD = 1$. $CE = 1.8$, $CF = 3$, $CG = 3.6$, $CH = 4.5$, and $CA = 9$. Which segment bisects angle $\angle ABC$?



(**A**) $\overline{BD}$ (**B**) $\overline{BE}$ (**C**) $\overline{BF}$ (**D**) $\overline{BG}$ (**E**) $\overline{BH}$